A NEW APPROACH OF ITERATIVE LEARNING CONTROL STRATEGY TOWARDS TUNING OF PI CONTROLLER IN A SPHERICAL TANK LEVEL PROCESS

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ABSTRACT
This paper provides the design and implementation of an Enhanced Iterative Learning control Strategy (EILCS) in a spherical tank level process. The dynamics of the process are described by differential equation and worst case model parameters are identified by influencing the step test technique. By means relay feedback technique, the periodic reference signal is generated and utilized as the input of EILCS loop. From the input and output chattering signals of the EILCS, optimized PI controller parameters are identified using Recursive Least Squares (RLS) fitting technique. The simulation results are furnished to illustrate the effectiveness of proposed method. Robustness is also analyzed.

Keywords: PI controller, ILC, RLS, ZPETC

1. INTRODUCTION
Being easy, robust, effective and suitable to a broad class of systems, PI (Proportional-Integral) controllers have been the most widely used and well known controllers in the many industries. Likewise, tuning of PI controllers have been of major interest to process control engineers for a long time since it helps a lot with the efforts in commissioning and operation of systems. Many tuning approaches have evolved in tuning the controller since 1942 when Ziegler and Nichols (ZN) [1] pioneered a unified systematic tuning approach in tuning the PID controller. In 1984, Astrom and Hagglund [2] introduced the relay-feedback test which arguably marked the coming of PID auto-tuning.

There are several schemes reported which are targeted at tuning of the PI [3-5]. Recently, a new and alternative approach is developed to tune a Smith predictor based on a Repetitive Control (RC) [6] methodology. Based on learning from previous repetitions, RC is able to improve system performance in subsequent ones and to yield a learned and optimized set of control signals to track a repetitive reference signal. The main idea associated with this paper is to use EILCS as a mechanism to derive the ideal control signal for processes with significant dead time to track a periodic reference sequence. This reference sequence, in the case of the usual ILC applications to robotics and motion systems where there is little time delay, can be the natural repetitive signal for the control system to execute the repetitive operations. In the case of process control applications, the frequency of the reference and repetitive sequence can be chosen to be at the ultimate frequency. This frequency can be efficiently obtained through a relay feedback experiment. In order for the ILC scheme to be applicable to processes with long dead time, the basic form is modified by adding a time-delay block to the feedback path. After the learning process has converged, a set of optimum control signals would be available for tuning of the PI controller. The recursive least squares algorithm is used, in this paper, for the controller parameter estimation of PI through signals fitting. The ZN based PI settings provide an initial parameter vector for the fitting algorithm.

The main contributions of the work presented in this paper are precisely simulation study of Enhanced Iterative Learning Control Strategy (EILCS) in a spherical tank level process and analysis of the performance criterion. In section 2 the process description of spherical tank is summarized. The design and structure of Enhanced Iterative Learning Control Strategy is detailed in section 3 Simulation results are analyzed in section 4. Finally, section 5 is summing up of the entire work.

2. PROCESS DYNAMICS
The spherical tank level system is shown in Figure 1. Here the control input \( f_{in} \) is being the input flow rate \( (m^3/s) \) and the output is \( x \) which is the fluid level \( (m) \) in the spherical tank.
Let, \( r \) = radius of tank
\( d_0 \) = thickness (diameter) of pipe (m) and initial height
\( r_{\text{surface}} \) = radius on the surface of the fluid varies according to the level (height) of fluid in the tank.
Dynamic model of tank is given as

\[
\frac{\delta}{\delta t} \left[ \int_{0}^{x} A(x) \, dx \right] = f_{in}(t) - a \sqrt{2gx}
\]
Where \( A(x) = \pi (2rx - x^2) \)

\[
a = \pi \left( \frac{d_0}{2} \right)^2
\]
Re write of dynamic model of tank at time \( t + \delta t \)

\[
A(x) \, dx = f_{in} \, \delta t - a \sqrt{2g(x - d_0)} \, \delta t
\]
By combining equation (1) to (4) we have

\[
\frac{\delta x}{\delta t} = \frac{\sin \delta t - \frac{\pi d_0^2}{2} \sqrt{2g(x - d_0)}}{\pi (2rx - x^2)}
\]

\[
\lim_{\delta t \to 0} \frac{\delta x}{\delta t} = \frac{dx}{dt}
\]
Therefore

\[
\frac{dx}{dt} = \frac{\sin \delta t - \frac{\pi d_0^2}{2} \sqrt{2g(x - d_0)}}{\pi (2rx - x^2)}
\]
Equation (6) shows the dynamic model of the spherical tank system

3. IDENTIFICATION OF MODEL PARAMETERS AND CONTROLLER SETTINGS
Initially the spherical tank liquid level system is maintained at a steady state of different operating point of 20%, 40%, 60% and 80%. A step magnitude of 5% level for each operating point is given in the system. The variation of level against time for each operating point is recorded separately until a new steady state is attained. From the level vs time data, the model parameters such as process gain \( (K_p) \) time constant \( (\tau_t) \) and delay \( (\tau_d) \) are computed. Among the model parameters obtained from different operating points, the worst case model parameters
such as larger process gain ($K_p$) smaller time constant ($\tau_p$) and larger delay ($\tau_d$) are considered here. The identified worst case model parameters for the system is given as

$$G(s) = \frac{1.76}{96.45s + 1} e^{-17.85s}$$

(7)

Based on these model parameters, PI mode controller settings ($K_c = 2.763$ and $K_i = 0.046$) is obtained by considering Z-N open loop tuning rule (ZNTR).

4. ENHANCED ITERATIVE LEARNING CONTROL STRATEGY (EILCS)

Iterative Learning Control is a model-free approach to achieve a superior system performance of repetitive systems over a finite time interval. The ILC is proposed by Arimoto et al. by which complete tracking performance is achieved as the given task is imposed iteratively. The Figure 2 shows the simple ILCS structure. To enhance the tracking performance of the simple ILCS, a Q filter is added with learning filter in the existing control loop as shown in Figure 3. This structure makes the system stable and enhances the overall performance. While this configuration works well for robotic and servo control applications with a moderately small time delay, it fails in the area of process control applications and requirements due to the typical presence of large time delay and large phase lag. To meet out the above said problems, the basic form of ILC is modified by adding a time delay block to the feedback path as shown in Figure 4.

The relay feedback configuration as shown in figure 4 is first applied to the process to obtain the repetitive excitation signal. Then the process is switched to ILC mode. Since the ultimate frequency, $y$ and $u$ is out of phase by $\pi$, the additional delay block $e^{-T/2}s$ (where $T$ is period of reference input) is introduced to align the phase of $\bar{e}$ and $u$ to linger in phase and thereby keeping the legitimate of ILC even in the occurrence of large delay. The suitable tracking performance is accomplished through the Iterative Learning Control Strategies, the signals $W$ (error signal) and $U$ (control signal) are attained and exercised to find the optimum PI controller parameters by using recursive least square algorithm (RLS). Here P-type update law is adopted for the ILC.
4.1 Design of the Key factors
To design L filter assume: For convergence analysis a small-gain type of argument is used, for which \( r_k = 0 \), and all initial conditions are zero.

Step 1: From Figure 3,

\[ e_k = \frac{-g}{1+gc} u_k \]  

(8)

Step 2: Adapt the learning up-date rule is:

\[ u_{k+1} = Q \cdot u_k + Q \cdot L \cdot e_k \]  

(9)

Step 3: Eliminate u:

\[ e_{k+1} = \frac{-g}{1+gc} u_{k+1} \]  

(10)

\[ e_{k+1} = \frac{-g}{1+gc} \cdot [Q \cdot u_k + Q \cdot L \cdot e_k] \]  

(11)

\[ = Q \left( 1 + \frac{-g}{1+gc} \cdot L \right) e_k \]  

(12)

\[ e_{k+1} = Q \left( 1 + \frac{-g}{1+gc} \cdot L \right) e_k \]  

(13)

It shows the propagation of the error signal from run to run. Convergence take place if

\[ \left| Q \left( 1 - \frac{e}{1+gc} \right) \right| < 1 \]  

(14)

Step 4: Infact, a suitable choice for L would be \( L = \frac{1+gc}{g} \). The inverse of L is nothing but the process-sensitivity

\[ T = \frac{g}{1+gc} \]  

i.e \( L = T^{-1} \). Due to the unstability and non-proper characterisitics of inverse complementary sensitivity, L can not be act as a filter. This problem is overcome by adapting Zero Phase Error Tracking Controller (ZPETC) algorithm [7]. The evaluation of ZPETC method is done by comparing bode plot of the original inverse complementary sensitivity \( T^{-1} \) and the approximated inverse complementary sensitivity ‘L’ (from ZPETC). It seems that the magnitude and phase plots of both cases are the same. It discloses that ‘L’ is the approximated inverse of
plant model. In this the phase plot, the phase caused by the delay has been taken into account. Figure 5 shows the bode plot of Inverse Complementary sensitivity (Original vs ZPETC).

4.2 Design of Robustness Filter (Q)
In practice, there may be an insignificant deviation of the developed process model from the actual process. This deviation leads the L filter to cause some disturbances in stability condition of the control loop for high frequencies. To overcome this problem, the robustness filter is included in the control loop. A first order continuous time low pass filter is considered here. i.e \( Q(s) = \frac{\omega_c}{s+\omega_c} \), where \( \omega_c \) is the cut-off frequency in rad/sec. The cut-off frequency is obtained from the Bode plot of the spherical tank system and it is found to be 0.01 as shown in Figure 6.

5. RECURSIVE LEAST SQUARE ESTIMATION (RLS) ALGORITHM [8]
The RLS fitting method is applied to the input and output chattering signals of the EILCS-relay construct to yield the gains of the optimized PI controller. The PI controller is described by

\[
    u(t) = k_c e + k_i \int_0^t e \, dt
\]

The above equation can be written in a matrix form as

\[
    u(t) = \begin{bmatrix} e \\ \int_0^t e \end{bmatrix} \begin{bmatrix} k_c \\ kl \end{bmatrix}
\]
The equation 8 is written in the linear in the parameters form

\[ u(t) = \theta(t) \phi^T \] (17)

Where

\[ \theta(t) = \begin{bmatrix} k_c \\ k_i \end{bmatrix} \text{ and } \phi^T = \begin{bmatrix} e \\ \int_0^t e \end{bmatrix} \] (18)

The RLS algorithm with a time varying forgetting factor can be directly used here as \( \alpha \) and \( T \) are available, the update of \( \theta(t) \) can be expressed as

\[ \theta(t) = \theta(t-1) + k(t)e(t) \] (19)

where \( \theta(t-1) \) refers to the controller settings identified during the last cycle, \( e(t) \) and \( k(t) \) are the error signal and Kalman gain vector, where

\[ e(t) = u(t) - \phi^T \theta(t-1) \] (20)
\[ k(t) = p(t-1)\phi(\phi^T p(t-1)\phi)^{-1} \] (21)
\[ p(t) = [1-k(t)\phi^T]p(t-1)/\lambda \] (22)

where \( \lambda \) is a forgetting factor (0 < \( \lambda \) < 1). There are two matrices to be initialized for the recursive algorithm and \( p(0) \) and \( \theta(0) \). It is usual to initialize \( p(0) \) such that \( p(0) = \alpha I \), where \( \alpha \) is a large number (10\(^{-4}\) – 10\(^{-6}\)) and \( I \) is the identity matrix. \( \theta(0) \) is set to be the gains of the PI controller before tuning. The robust control configuration, comprising of the relay and the EILC controller, puts a high gain in the loop and ensures satisfactory closed-loop performance. Although it incurs a chattering phenomenon, the chattering signals are used to tune PI controller parameters.

6. RESULTS AND DISCUSSION

To analyze the effectiveness of the EILCS, design parameters such as learning filter \( L \) and robustness filter \( Q \) are designed initially by considering the spherical tank level process model equation 7 and it is given by

\[ L = \frac{-2000s + 119.6}{s + 20} \quad Q = \frac{0.01}{s + 0.01} \]

Besides, ‘\( k_l \)’ is learning gain and it is chosen as 0.1(0 < \( k_l \) < 1). The relay feedback configuration as shown in figure.3, is first applied to the process to get the repetitive excitation signal for the EILCS as shown in Figure.7. Then the process is switched to EILCS set up with the repetitive excitation signal. Figure.8 and Figure.9 shows the reference input \( r \) and process output \( y \) of simple ILCS and EILCS which are 180 degree out of phase. After tracking performance is consummate through the Iterative Learning Control Strategies, the signals \( W \) and \( U \) are attained (refer Figure 10 and 11). By using recursive least square algorithm, the signals \( W \) and \( U \) are exercised to find the optimum PI controller parameters (\( K_c = 2.911; K_i = 0.0456 \)). Likewise, PI controller settings for simple ILCS are computed. Controller parameters for cases are reported in Table 1.

<table>
<thead>
<tr>
<th>Table 1. PI Controller Parameters</th>
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<tbody>
<tr>
<td>Control loop</td>
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<tr>
<td>Conventional PI</td>
</tr>
<tr>
<td>EILCS</td>
</tr>
<tr>
<td>EILCS</td>
</tr>
</tbody>
</table>

Simulation run of spherical tank level process is carried out with EILCS based PI values. Initially the level in the tank is maintained at 40 % of operating level. After that, a step size of ±5% of level is applied to control loop. Similar test runs of simple ILCS based PI and ZN based PI is executed and the responses of all the three cases are recorded in Figure 12 and Figure13. From the results, the performances of each control scheme are analyzed in terms of ISE and IAE and performance indices are tabulated in Table 2. The results clearly prove that EILCS based PI controller dominates the other controllers. To test the robustness of the EILCS based PI controller, the operating point is changed to 60% and same procedure is repeated. The responses are traced in Figure 14 and 15 and their performance indices are tabulated in Table 3. It is quite apparent that the EILCS based PI controller outperforms than the other controllers and also shows its heftiness.
Figure 7. Output response of the process under relay feedback

Figure 8. Input and Output of the ILCS structure

Figure 9. Input and Output of the EILCS structure
Iterative Learning Control Strategy

**Figure 10.** Signals $U$ and $W$ based on ILCS structure

**Figure 11.** Signals $U$ and $W$ based on EILCS structure

**Figure 12.** Step responses of different controllers at operating range of 40% with 5% step size
Figure.13. Step responses of different controllers at operating range of 40% with -5% step size

Figure.14. Step responses of different controllers at operating range of 60% with 5% step size

Figure.15. Step responses of different controllers at operating range of 60% with -5% step size
Table 2. Performance Indices at operating range of 40%

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>Step change (+5)</th>
<th>Step change (-5)</th>
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<tr>
<td></td>
<td>ISE</td>
<td>IAE</td>
</tr>
<tr>
<td>ZNPI</td>
<td>679</td>
<td>321</td>
</tr>
<tr>
<td>ILCS</td>
<td>636</td>
<td>303</td>
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<tr>
<td>EILCS</td>
<td>626</td>
<td>276</td>
</tr>
</tbody>
</table>

Table 3. Performance Indices at operating range of 60%

<table>
<thead>
<tr>
<th>CONTROLLER</th>
<th>Step change (+5)</th>
<th>Step change (-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ISE</td>
<td>IAE</td>
</tr>
<tr>
<td>ZNPI</td>
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<td>322</td>
</tr>
<tr>
<td>ILCS</td>
<td>652</td>
<td>303</td>
</tr>
<tr>
<td>EILCS</td>
<td>641</td>
<td>275</td>
</tr>
</tbody>
</table>

7. CONCLUSION
In this paper, an Enhanced ILC based PI control structure is developed and put into operation in the spherical tank level process. The mathematical model of spherical tank level process is described by differential equation and worst case model parameters are identified by influencing the step test technique. By using relay feedback technique, the periodic reference signal is generated and utilized as the input of EILCS control loop. From the input and output signals of the EILCS, the optimized PI controller parameters are identified using RLS fitting technique. Finally, the EILCS-based PI parameters are executed in the spherical tank level process and the performances are analyzed. A comparison of this structure with other control strategies such as conventional and simple ILC is also made in this work. The simulation results are furnished to illustrate the efficiency of EILCS-based PI approach. This control structure is more appropriate to the system having a large delay.

8. REFERENCES
[1] Ziegler, J.B.; and Nichols, N. B. Optimum settings for automatic controllers, ASME Transactions, 64, 759-768. (1942)