ON COMMON FIXED POINT THEOREMS FOR SEMI-COMPATIBLE AND OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN MENERG SPACE

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**ABSTRACT**

In this paper, the concept of semi-compatibility and occasionally weak compatibility in Menger space has been applied to prove a common fixed point theorem for six self maps. Our result generalizes and extends the result of Pathak and Verma [9].

**Keywords:** Probabilistic metric space, Menger space, common fixed point, compatible maps, semi-compatible maps, weak compatibility.

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1. **INTRODUCTION**

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [7]. It is a probabilistic generalization in which we assign to any two points \(x\) and \(y\), a distribution function \(F_{x,y}\). Schweizer and Sklar [11] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [12] obtained a generalization of Banach Contraction Principle on a complete Menger space which is a milestone in developing fixed-point theory in Menger space.

Recently, Jungck and Rhoades [6] termed a pair of self maps to be coincidentally commuting or equivalently weakly compatible if they commute at their coincidence points. Sessa [13] initiated the tradition of improving commutativity in fixed-point theorems by introducing the notion of weak commuting maps in metric spaces. Jungck [5] soon enlarged this concept to compatible maps. The notion of compatible mapping in a Menger space has been introduced by Mishra [8]. In the sequel, Pathak and Verma [9] proved a common fixed point theorem in Menger space using compatibility and weak compatibility. Using the concept of compatible mappings of type (A), Jain et. al. [2, 3] proved some interesting fixed point theorems in Menger space. Afterwards, Jain et. al. [4] proved the fixed point theorem using the concept of weak compatible maps in Menger space.


In this paper a fixed point theorem for six self maps has been proved using the concept of semi-compatible maps and occasionally weak compatibility which turns out to be a material generalization of the result of Pathak and Verma [9].

2. **Preliminaries.**

**Definition 2.1.** A mapping \(F : \mathbb{R} \to \mathbb{R}^+\) is called a *distribution* if it is non-decreasing left continuous with\
\[
\inf \{ F(t) \mid t \in \mathbb{R} \} = 0 \quad \text{and} \quad \sup \{ F(t) \mid t \in \mathbb{R} \} = 1.
\]

We shall denote by \(L\) the set of all distribution functions while \(H\) will always denote the specific distribution function defined by\
\[
H(t) = \begin{cases} 
0, & t \leq 0 \\
1, & t > 0
\end{cases}
\]

**Definition 2.2.** [8] A mapping \(t : [0, 1] \times [0, 1] \to [0, 1]\) is called a *\(t\)-norm* if it satisfies the following conditions:

\[
\begin{align*}
(t-1) & \quad t(a, 1) = a, \quad t(0, 0) = 0; \\
(t-2) & \quad t(a, b) = t(b, a); \\
(t-3) & \quad t(c, d) \geq t(a, b); \quad \text{for } c \geq a, d \geq b.
\end{align*}
\]
(t-4) \[ t(t(a, b), c) = t(a, t(b, c)) \] for all \( a, b, c, d \in [0, 1] \).

**Definition 2.3.** [8] A probabilistic metric space (PM-space) is an ordered pair \((X, F)\) consisting of a non empty set \(X\) and a function \(F : X \times X \to L\), where \(L\) is the collection of all distribution functions and the value of \(F\) at \((u, v) \in X \times X\) is represented by \(F_{u,v}\). The function \(F_{u,v}\) assumed to satisfy the following conditions:

1. **(PM-1)** \(F_{u,v}(x) = 1\), for all \(x > 0\), if and only if \(u = v\);
2. **(PM-2)** \(F_{u,v}(0) = 0\);
3. **(PM-3)** \(F_{u,v} = F_{v,u}\);
4. **(PM-4)** If \(F_{u,v}(x) = 1\) and \(F_{v,w}(y) = 1\) then \(F_{u,w}(x + y) = 1\), for all \(u, v, w \in X\) and \(x, y > 0\).

**Definition 2.4.** [8] A Menger space is a triplet \((X, F, t)\) where \((X, F)\) is a PM-space and \(t\) is a \(t\)-norm such that the inequality

\[ \text{(PM-5)} \quad F_{u,w}(x + y) \geq t\{F_{u,v}(x), F_{v,w}(y)\}, \quad \text{for all} \quad u, v, w \in X, x, y \geq 0. \]

**Proposition 2.1.** [8] If \((X, d)\) is a metric space then the metric \(d\) induces mappings \(F : X \times X \to L\), defined by

\[ F_{p,q}(x) = H(x - d(p, q)), \quad p, q \in X, \]

where \(H(k) = 0\), for \(k \leq 0\) and \(H(k) = 1\), for \(k > 0\). Further if, \(t : [0,1] \times [0,1] \to [0,1]\) is defined by \(t(a, b) = \min \{a, b\}\). Then \((X, F, t)\) is a Menger space. It is complete if \((X, d)\) is complete.

**Definition 2.5.** [8] A sequence \(\{x_n\}\) in a Menger space \((X, F, t)\) is said to be convergent and converges to a point \(x\) in \(X\) if and only if for each \(\varepsilon > 0\) and \(\lambda > 0\), there is an integer \(M(\varepsilon, \lambda)\) such that \(F_{x_n, x}(\varepsilon) > 1 - \lambda\), for all \(n \geq M(\varepsilon, \lambda)\).

Further the sequence \(\{x_n\}\) is said to be Cauchy sequence if for \(\varepsilon > 0\) and \(\lambda > 0\), there is an integer \(M(\varepsilon, \lambda)\) such that \(F_{x_n, x_m}(\varepsilon) > 1 - \lambda\), for all \(m, n \geq M(\varepsilon, \lambda)\). A Menger PM-space \((X, F, t)\) is said to be complete if every Cauchy sequence in \(X\) converges to a point in \(X\).

**Definition 2.6.** [9] Self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are said to be weak compatible if they commute at their coincidence points i.e. \(Ax = Sx\) for \(x \in X\) implies \(ASx = SAx\).

**Definition 2.7.** [9] Self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are said to be compatible if \(F_{ASx_n, SAx_n}(x) \to 1\) for all \(x > 0\), whenever \(\{x_n\}\) is a sequence in \(X\) such that \(Ax_n, Sx_n \to u\) for some \(u \in X\), as \(n \to \infty\).

**Definition 2.8.** [14] Self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are said to be semi-compatible if \(F_{ASx_n, Sx_n}(x) \to 1\) for all \(x > 0\), whenever \(\{x_n\}\) is a sequence in \(X\) such that \(Ax_n, Sx_n \to u\), for some \(u \in X\), as \(n \to \infty\).

**Definition 2.9.** Self maps \(A\) and \(S\) of a N.A. Menger PM-space \((X, F, t)\) are said to be occasionally weakly compatible (owc) if and only if there is a point \(x\) in \(X\) which is coincidence point of \(A\) and \(S\) at which \(A\) and \(S\) commute.

**Example 2.1.** Let \((X, F, t)\) be the Menger PM-space, where \(X = [0, 4]\) Define \(F\) by

\[ F_{x, y}(t) = \begin{cases} \frac{t}{t + |x - y|} & \text{if } t > 0, \\ 0, & \text{if } t = 0 \end{cases} \]

Define \(A, S : X \to X\) by

\[ Ax = 4x \text{ and } Sx = x^2 \text{ for all } x \in X \text{ then } Ax = Sx \text{ for } x = 0 \text{ and } 4. \]

But \(AS(0) = SA(0)\) and \(AS(4) \neq SA(4)\).
Thus, S and T are occasionally weakly compatible mappings but not weakly compatible.

**Remark 2.1.** In view of above example, it follows that the concept of occasionally weakly compatible is more general than that of weak compatibility.

**Lemma 2.1.** [9] Let \((X, F, \ast)\) be a Menger space with t-norm \(*\) such that the family \(\{*_n(x)\}_{n \in \mathbb{N}}\) is equicontinuous at \(x = 1\) and let \(E\) denote the family of all functions \(\phi: \mathbb{R}^+ \to \mathbb{R}^+\) such that \(\phi\) is non-decreasing with \(\lim_{n \to \infty} \phi^n(t) = +\infty, \quad \forall \ t > 0\). If \(\{y_n\}_{n \in \mathbb{N}}\) is a sequence in \(X\) satisfying the condition

\[
F_{y_{n+1}}(t) \geq F_{y_{n+1}, y_n}(\phi(t)),
\]

for all \(t > 0\) and \(\alpha \in [-1, 0]\), then \(\{y_n\}_{n \in \mathbb{N}}\) is a Cauchy sequence in \(X\).

### 3. MAIN RESULT

**Theorem 3.1.** Let \(A, B, S, T, P\) and \(Q\) be self maps of a complete Menger space \((X, F, \ast)\) with \(* = \min\) satisfying:

1. \((P, AB)\) is semi-compatible and \((Q, ST)\) is occasionally weak compatible;
2. \(\alpha > 0\) for all \(x, y \in X, t > 0\) and \(\phi \in E\).

Then \(A, B, S, T, P\) and \(Q\) have a unique common fixed point in \(X\).

**Proof.** Suppose \(x_0 \in X\). From condition (3.1.1) \(\exists \ x_1, x_2 \in X\) such that

\[ P_{x_0} = ST_{x_1} \quad \text{and} \quad Q_{x_1} = AB_{x_2}. \]

Inductively, we can construct sequences \(\{x_n\}\) and \(\{y_n\}\) in \(X\) such that

\[ y_{2n} = P_{x_{2n}} = ST_{x_{2n+1}} \quad \text{and} \quad y_{2n+1} = Q_{x_{2n+1}} = AB_{x_{2n+2}} \]

for \(n = 0, 1, 2, \ldots\).

**Step 1.** Let us show that \(F_{y_{n+2}}(t) \geq F_{y_{n+1}, y_n}(\phi(t)).

For, putting \(x_{2n+2}\) for \(x\) and \(x_{2n+1}\) for \(y\) in (3.1.5) and then on simplification, we have

\[ [1 + \alpha F_{ABx_{2n+2}, STx_{2n+1}}(t)] \ast F_{Px_{2n+2}, Qx_{2n+1}}(t) \]

\[
\geq \alpha \min\{F_{Px_{2n+2}, ABx_{2n+2}}(t), F_{Qx_{2n+1}, STx_{2n+1}}(t), F_{Px_{2n+2}, STx_{2n+1}}(2t) \}
\]

\[
\geq \alpha \min\{F_{y_{2n+2}, y_{2n+1}}(t), F_{y_{2n+1}, y_{2n}}(t), F_{y_{2n+2}, y_{2n}}(2t) \}
\]

\[
\geq \alpha \min\{F_{y_{2n+2}, y_{2n+1}}(t), F_{y_{2n+1}, y_{2n}}(t), F_{y_{2n+2}, y_{2n}}(2t) \}
\]

\[
\geq \alpha \min\{F_{y_{2n+2}, y_{2n+1}}(t), F_{y_{2n+1}, y_{2n}}(t), F_{y_{2n+2}, y_{2n}}(2t) \}
\]

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Case I. Suppose $P$ is continuous.

As $P$ is continuous and $(P, AB)$ is semi-compatible, we get

$$PABx_{2n+2} \rightarrow Pz \quad \text{and} \quad PABx_{2n+2} \rightarrow ABz.$$  \hfill (3.1.7)

Since the limit in Menger space is unique, we get

$$Pz = ABz.$$  \hfill (3.1.8)

**Step II.** We prove $Pz = z$. Put $x = z, y = x_{2n+1}$ in (3.1.5) and let $Pz \neq z$. Then

$$[1 + \alpha F_{ABz}(t)] \cdot F_{Pz}(t) \geq \alpha \min\{F_{Pz}(t), F_{ABz}(t), F_{STx_{2n+1}}(t), F_{STx_{2n+1}}(t) \} + F_{ABz}(t).$$

**Step III.** We prove $Pz = z$. Put $x = z, y = x_{2n+1}$ in (3.1.5) and let $Pz \neq z$. Then

$$[1 + \alpha F_{ABz}(t)] \cdot F_{Pz}(t) \geq \alpha \min\{F_{Pz}(t), F_{ABz}(t), F_{STx_{2n+1}}(t), F_{STx_{2n+1}}(t) \} + F_{ABz}(t).$$
Letting \(n \to \infty\) and using (3.1.6) and (3.1.8), we get
\[
[1 + \alpha Pz, z(t)] * F_{Pz}, z(t)
\geq \alpha \min\{F_{Pz}, Pz(t) * Fz, z(t), F_{Pz}, z(2t) * Fz, Pz(2t)\} + F_{Pz}, z(\phi(t)) * F_{Pz}, Pz(\phi(t))
\]
\[
* F_{Pz}, z(\phi(t)) * F_{Pz}, z(2\phi(t)) * F_{Pz}, Pz(2\phi(t))
\]
\[
F_{Pz}, z(t) + \alpha F_{Pz}, z(t) * F_{Pz}, z(t)
\geq \alpha \min\{1 \cdot 1, F_{Pz}, z(2t) \cdot F_{Pz}, z(2t)\}
\]
\[
* F_{Pz}, z(\phi(t)) * F_{Pz}, z(2\phi(t))
\]
which is a contradiction and hence, \(Pz = z\) and so \(z = Pz = ABz\).

**Step III.** Put \(x = Bz\) and \(y = x_{2n+1}\) in (3.1.5), we get
\[
[1 + \alpha F_{ABBz}, STx_{2n+1}(t)] * F_{Pbz}, Qx_{2n+1}(t)
\geq \alpha \min\{F_{Pbz}, ABBz(t) * F_{Qx_{2n+1}}, STx_{2n+1}(t), F_{Pbz}, STx_{2n+1}(2t) * F_{Qx_{2n+1}}, ABBz(2t)\}
\]
\[
+ F_{ABBz}, STx_{2n+1}(\phi(t)) * F_{Pbz}, ABBz(\phi(t)) * F_{Qx_{2n+1}}, STx_{2n+1}(\phi(t))
\]
\[
* F_{Pbz}, STx_{2n+1}(2\phi(t)) * F_{Qx_{2n+1}}, ABBz(2\phi(t))
\]
As \(BP = PB, AB = BA\) so we have
\(P(Bz) = B(Pz) = Bz\) and \(AB(Bz) = B(AB)z = Bz\).

Letting \(n \to \infty\) and using (3.1.6), we get
\[
[1 + \alpha F_{Bz}, z(t)] * F_{Bz}, z(t)
\geq \alpha \min\{F_{Bz}, Bz(t) * Fz, z(t), F_{Bz}, z(2t) * Fz, Bz(2t)\}
\]
\[
+ F_{Bz}, z(\phi(t)) * F_{Bz}, Bz(\phi(t)) * F_{Bz}, z(2\phi(t)) * F_{Bz}, Bz(2\phi(t))
\]
\[
F_{Bz}, z(t) + \alpha F_{Bz}, z(t) * F_{Bz}, z(t)
\geq \alpha \min\{1 \cdot 1, F_{Bz}, z(2t)\}
\]
\[
* F_{Bz}, z(\phi(t)) * F_{Bz}, z(2\phi(t))
\]
which is a contradiction and we get \(Bz = z\) and so \(z = ABz = Az\).

Therefore, \(Pz = Az = Bz = z\). (3.1.9)

**Step IV.** Since \(P(X) \subseteq ST(X)\) there exists \(u \in X\) such that
\(z = Pz = STu\).
Put \(x = x_{2n}\) and \(y = u\) in (3.1.5), we get
\[ [1 + \alpha F_{ABx_{2n}}, STu(t)] * F_{Px_{2n}}, Qu(t) \]
\[ \geq \min_{\alpha F_{ABx_{2n}}, STz(t), F_{Px_{2n}}, STz(2t) * F_{Qz}, ABx_{2n}(2t)} \]
\[ + F_{ABx_{2n}}, STz(t), F_{Px_{2n}}, ABx_{2n}(\phi(t)) * F_{Qz}, STz(t), F_{Px_{2n}}, STz(2\phi(t)) \]
\[ * F_{Qz}, ABx_{2n}(2\phi(t)). \]

Letting \( n \to \infty \) and using \((3.1.6)\), we get
\[ [1 + \alpha F_{z, z(t)}] * F_{z, Qu(t)} \]
\[ \geq \min_{F_{z, z(t)}, F_{z, z(2t)] * F_{Qz}, z(\phi(t))] + F_{z, z(\phi(t))] * F_{z, z(2\phi(t))] \}
\[ * F_{Qz}, ABx_{2n}(2\phi(t)). \]

which is a contradiction by lemma \((2.1)\) and we get
\( Qu = z \) and so \( Qu = z = STu \).
Since \((Q, ST)\) is occasionally weak-compatible, we have
\( STQu = QSTu \) i.e. \( STz = Qz \).

**Step V.** Put \( x = x_{2n} \) and \( y = z \) in \((3.1.5)\), we have
\[ [1 + \alpha F_{ABx_{2n}}, STz(t)] * F_{Px_{2n}}, Qz(t) \]
\[ \geq \min_{F_{z, z(t)}, F_{z, z(2t)] * F_{Qz, STz(t), F_{Px_{2n}}, STz(2t) * F_{Qz}, ABx_{2n}(2t)] \}
\[ + F_{ABx_{2n}}, STz(t), F_{Px_{2n}}, ABx_{2n}(\phi(t)) * F_{Qz, STz(t), F_{Px_{2n}}, STz(2\phi(t)) \]
\[ * F_{Qz}, ABx_{2n}(2\phi(t)). \]

Letting \( n \to \infty \) and using \((3.1.6)\) and step IV, we get
\[ [1 + \alpha F_{z, Qz(t)}] * F_{z, Qz(t)} \]
\[ \geq \min_{F_{z, z(t), F_{z, z(2t)] * F_{Qz, z(\phi(t))] + F_{z, z(\phi(t))] * F_{z, z(2\phi(t))] \}
\[ * F_{Qz, Qz(\phi(t))] * F_{z, Qz(2\phi(t))] * F_{Qz, Qz(2\phi(t))} \]
\[ * F_{Qz, ABx_{2n}(2\phi(t)). \]

which is a contradiction and we get \( Qz = z \).

**Step VI.** Put \( x = x_{2n} \) and \( y = Tz \) in \((3.1.5)\), we have
\[ [1 + \alpha F_{ABx_{2n}}, STz(t)] * F_{Px_{2n}}, QTz(t) \]
\[ \geq \min_{F_{z, z(t), F_{z, z(2t)] * F_{QTz, STz(t), F_{Px_{2n}}, STz(2t) * F_{QTz, ABx_{2n}(2t)] \}
\[ + F_{ABx_{2n}}, STz(t), F_{Px_{2n}}, ABx_{2n}(\phi(t)) * F_{QTz, STz(t), F_{Px_{2n}}, STz(2\phi(t)) \]
\[ * F_{QTz, ABx_{2n}(2\phi(t)). \]

As \( QT = TQ \) and \( ST = TS \), we have
QTz = TQz = Tz and ST(Tz) = T(S(Tz)) = Tz.

Letting n → ∞, we get

\[ [1 + \alpha F_{z, Tz}(t)] * F_{z, Tz}(t) \geq \alpha \min \{F_{z}, z(t) * F_{z, Tz}(t), F_{z}, Tz(2t) * F_{z, Tz}(t), F_{z}, Tz(2t) * F_{z}(\phi(t)) * F_{z, Tz}(t) * F_{z}(\phi(t)) \]  

which is a contradiction and we get Tz = z.

Now, STz = Tz = z implies Sz = z.

Hence, Sz = Tz = Qz = z.  \hspace{1cm} (3.1.10)

Combining (3.1.9) and (3.1.10), we get

Az = Bz = Pz = Qz = Sz = Tz = z

i.e. z is a common fixed point of A, B, P, Q, S and T.

Case II. Suppose AB is continuous.

Since AB is continuous and (P, AB) is semi-compatible, we get

\[ (AB)^2 x_{2n} → ABz, \quad PABx_{2n} → ABz. \hspace{1cm} (3.1.11) \]

Now, we prove ABz = z.

Step VII. Put x = ABx_{2n} and y = x_{2n+1} in (3.1.5) and assuming ABz ≠ z, we get

\[ [1 + \alpha F_{ABABz, 2n+1}(t)] * F_{PABx_{2n+1}}(t) \geq \alpha \min \{F_{PABx_{2n+1}}, ABABx_{2n+1}(t) * F_{Qx_{2n+1}}, STx_{2n+1}(t), F_{PABx_{2n+1}}, STx_{2n+1}(2t) \]  

\[ * F_{Qx_{2n+1}}, ABABx_{2n+1}(2t) * F_{PABx_{2n+1}}, ABABx_{2n+1}(\phi(t)) \]  

\[ * F_{Qx_{2n+1}}, STx_{2n+1}(\phi(t)) * F_{PABx_{2n+1}}, STx_{2n+1}(2\phi(t)) * F_{Qx_{2n+1}}, ABABx_{2n+1}(2\phi(t)). \]

Letting n → ∞ and using (3.1.11), we get

\[ [1 + \alpha F_{ABz, z(t)}] * F_{ABz, z(t)} \geq \alpha \min \{1, F_{ABz, z(\phi(t))} * F_{ABz, z(\phi(t))} \]  

\[ * F_{ABz, ABz(\phi(t))} * F_{z, ABz(\phi(t))} * F_{z, z(\phi(t))} * F_{z, ABz(\phi(t))} \]  

\[ F_{ABz, z(t)} \]  

\[ + \alpha F_{ABz, z(t)} \geq \alpha \min \{1, F_{ABz, z(\phi(t))} * F_{ABz, z(\phi(t))} \]  

\[ * F_{ABz, z(\phi(t))} * F_{z, z(\phi(t))} + F_{ABz, z(\phi(t))} \]  

\[ + F_{ABz, z(\phi(t))} + F_{ABz, z(\phi(t))} \]

which is a contradiction and we get ABz = z.

Step VIII. Put x = z and y = x_{2n+1} in (3.1.5), we get

\[ [1 + \alpha F_{ABz, STx_{2n+1}}(t)] * F_{Pz, Qx_{2n+1}}(t) \geq \alpha \min \{F_{Pz, ABz(t)} * F_{Qx_{2n+1}}, STx_{2n+1}(t), F_{Pz, STx_{2n+1}}, STx_{2n+1}(t), F_{Qx_{2n+1}}, ABz(2t) \]  

\[ + F_{ABz, STx_{2n+1}}, STx_{2n+1}(\phi(t)) * F_{Qx_{2n+1}}, STx_{2n+1}(\phi(t)) * F_{Pz, STx_{2n+1}}(\phi(t)) \]  

\[ + F_{ABz, STx_{2n+1}}, STx_{2n+1}(\phi(t)) * F_{Qx_{2n+1}}, STx_{2n+1}(\phi(t)) * F_{Pz, STx_{2n+1}}(\phi(t)) \]

\[ + F_{ABz, STx_{2n+1}}(\phi(t)) * F_{Pz, ABz(\phi(t))} + F_{ABz, z(\phi(t))} + F_{ABz, z(\phi(t))} \]

\[ + F_{ABz, z(\phi(t))} \]  

\[ + F_{ABz, z(\phi(t))} \]

which is a contradiction and we get ABz = z.
Corollary 3.1.

Let A, S, P and Q be self maps of a complete Menger space \((X, F, *)\) with \(* = \min\) satisfying:

(a) \(P(X) \subseteq S(X), \ Q(X) \subseteq A(X)\);
(b) either P or A is continuous;

(c) (P, A) is semi-compatible and (Q, S) is occasionally weak compatible;

(d) \[1 + \alpha F_{Ax, Sy(t)} \cdot F_{Px, Qy(t)} \geq \alpha \min\{F_{Px, Ax(t)} \cdot F_{Qy, Sy(t)}, F_{Px, Qy(2t)} \cdot F_{Qy, Ax(2t)}\} \]
\[+ \alpha F_{Ax, Sy(\phi(t))} + F_{Px, Ax(\phi(t))} + F_{Qy, Sy(\phi(t))} + F_{Px, Sy(2\phi(t))} + F_{Qy, Ax(2\phi(t))}\]

for all \(x, y \in X, t > 0\) and \(\phi \in E\).

Then A, S, P and Q have a unique common fixed point in X.

**Remark 3.2.** In view of remark 3.1, corollary 3.1 is a generalization of the result of Pathak and Verma [9] in the sense that condition of compatibility of the first pair of self maps has been restricted to semi-compatibility.

**REFERENCES**


