TIME DEPENDENT NONLINEAR FRACTIONAL SCHröDINGER EQUATION WITH TRAPPING POTENTIAL ENERGY DISTURBANCES 
AN EXPONENTIAL DECAY DISTURBANCE

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ABSTRACT
We paper present an investigation for the time-dependent unsteady state fractional nonlinear Schrödinger equation with trapping potential energy disturbances and an exponential decay disturbance. The approximate analytical solutions obtained by the homotopy analysis method (HAM) and the Adomian decomposition technique (ADM). The solutions are shown graphically and the effects of involved parameters on the probability density are analyzed.

Keywords: Schrödinger equation; fractional derivative; potential energy disturbance; homotopy analysis method; Adomian decomposition.

1. INTRODUCTION
Nonlinear Schrödinger equations (NLSE) are often encountered in many branches of science and engineering. A variety of analytical methods have been proposed for solving these equations [1-4]. The sharp asymptotic behavior of solutions in one space dimension, the stability of plane-wave solutions of a dissipative generalization of the nonlinear Schrödinger equation, and the self-similarity and asymptotic stability for coupled nonlinear Schrödinger equations in high dimensions are discussed [5-7].

In recent years, a considerable attention has been devoted to the fractional calculus [9-10]. The fractional Schrödinger equation is a basic equation of fractional quantum mechanics. It was first proposed by Nick Laskin as a result of extending the Feynman path integral, from the Brownian-like to Lévy-like quantum mechanical paths [11].

We consider in this paper a particle in the potential field near its equilibrium point with exponential decay disturbances. The time-dependent fractional Nonlinear Schrödinger equation is written as:

\[ i \frac{D^{\alpha} \Psi}{Dt^{\alpha}} = -\frac{\partial^2 \Psi}{\partial x^2} + |\Psi|^2 \Psi + \varepsilon \frac{\partial \Psi}{\partial x} \Psi + \beta e^{-x} \]

\[ \Psi(x,0) = f(x) \]

(1)

where \( \Psi \) is the unknown wave function, \( \frac{\partial^{\alpha} \Psi}{\partial t^{\alpha}} (\alpha \text{ that varies from 0 to 1}) \) is the Caputo’s fractional differential operator[9], \( \varepsilon \frac{\partial \Psi}{\partial x} \) is the trapping potential, \( \beta e^{-x} \) is the exponential decay disturbance, and \( \varepsilon \) and \( \beta \) are small disturbance parameter.

2. PRELIMINARIES AND NOTATIONS
Definition 1. The Riemann–Liouville integral operator of order \( \alpha > 0 \) is defined as:

\[ J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau \quad (\alpha > 0, t > 0) \]

\[ J^{0} f(t) = f(t) \]

(3)

(4)

Definition 2. The Caputo integral operator of order \( \alpha > 0 \) is defined as:
\[ D^\alpha f(t) = J^{n-\alpha} f^{(n)}(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau \] (5)

\[ (n-1 < \alpha < n, n \in N, t > 0) \]

\[ D^\alpha f(t) = f^{(n)}(t) \quad (\alpha = n, \ n \in N) \] (6)

Properties of the operators \( J \) and \( D \) can be found in [9], we mention only the following:

\[ J^\alpha D^\alpha f(t) = f(t) - \sum_{i=0}^{n-1} \frac{f^{(i)}(0^+)}{i!} t^i \quad (n-1 < \alpha \leq n, n \in N) \] (7)

\[ D^\alpha J^\alpha f(t) = f(t) \] (8)

3. SOLUTIONS OF HAM AND ADM METHOD

3.1. Solutions of HAM

The homotopy analysis method (HAM) is a new analytical method which was first proposed by Liao [12-13]. In this paper, we apply the homotopy analysis method (HAM) to the fractional nonlinear Schrödinger equation (1). Let us consider a fractional differential equation as follow:

\[ N[u(x,t)] = 0 \] (9)

where \( N \) is a fractional differential operator, \( x \) and \( t \) are the space and time independent variables, \( u(x,t) \) is an unknown function, respectively. Taking the \( q \in [0,1] \) as an embedded variable, one can construct the following equation:

\[ (1-p)L[\phi(x,t;p) - u_0(x,t)] = phH(x,t)N[\phi(x,t;p)] \] (10)

where \( h \) is a non-zero auxiliary parameter, \( H(x,t) \) is a non-zero auxiliary function, and \( L \) is an auxiliary linear operator which satisfies \( L[\phi(x,t)] = 0 \) when \( \phi(x,t) = 0 \). \( u_0(x,t) \) is the initial guess solution. There is great freedom to choose the auxiliary function \( H(x,t) \) and the auxiliary parameter \( h \) in homotopy analysis method.

When \( p = 0 \) and \( p = 1 \), it holds

\[ \phi(x,t;0) = u_0(x,t), \ \phi(x,t;1) = u(x,t) \] (11)

From the above we can see that as \( p \) increase from 0 to 1, the solution \( \phi(x,t;p) \) varies from the initial guess \( u_0(x,t) \) to the exact solution \( u(x,t) \). Expanding \( \phi(x,t;p) \) in Taylor series with respect to \( p \), yields

\[ \phi(x,t;p) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)p^m \] (12)

where

\[ u_m(x,t) = \left. \frac{\partial^m \phi(x,t;p)}{\partial p^m} \right|_{p=0} \] (13)

Thus, we obtain

\[ u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t) \] (14)

If the auxiliary linear operator \( L \), the initial guess \( u_0(x,t) \), the auxiliary function \( H(x,t) \) and the auxiliary parameter \( h \) are properly chosen, the series converges at \( p = 1 \).
Define the vector
\[ \tilde{u}_n = \{u_0(x,t), u_1(x,t), \ldots, u_n(x,t)\} \]  
(15)

Differentiating Eq.(10) \( m \) times with respect to the embedding parameter \( p \) and then setting \( p = 0 \), we obtain the \( m \) th-order deformation equation
\[ L\left[u_m(x,t) - \chi_m u_{m-1}(x,t)\right] = hH(x,t)R_m\left[u_{m-1}(x,t)\right] \]  
(16)

where
\[ R_m\left[u_{m-1}(x,t)\right] = \frac{1}{(m-1)!} \left. \frac{\partial^{m-1} N[\phi(x,t; p)]}{\partial p^{m-1}} \right|_{p=0} \]  
(17)

and
\[ \chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \]  
(18)

In this paper, we can choose the linear operator
\[ L[\phi(x,t)] = D_t^{\alpha}[\phi(x,t)] \]  
(19)

with the property \( L[C] = 0 \), where \( C \) is constant and define the nonlinear operator as
\[ N[\phi(x,t; p)] = D_t^{\alpha}\phi(x,t; p) - i\frac{\partial^2 \phi(x,t; p)}{\partial x^2} \]
\[ + i\left|\phi(x,t; p)\right|^2\phi(x,t; p) + i\varepsilon\phi(x,t; p)\frac{\partial \phi(x,t; p)}{\partial x} + i\beta e^{-x} \]  
(20)

In terms of complex function \( \phi(x,t; p) \), Eq. (20) can be rewritten as
\[ N[\phi(x,t; p)] = D_t^{\alpha}\phi - i\frac{\partial^2 \phi}{\partial x^2} + i\phi\overline{\phi} + i\varepsilon\phi\frac{\partial \phi}{\partial x} + i\beta e^{-x} \]  
(21)

Where \( \overline{\phi} \) is the conjugate of \( \phi \).

Using the above formula, setting the \( f(x) = e^{\lambda x}, \lambda < 0 \), we obtain
\[ \Psi_0(x,t) = e^{\lambda x} \]  
(22)

\[ \Psi_1 = iht^{\alpha} e^{3\lambda x} + \beta e^{-x} + \varepsilon \beta e^{2\lambda x} - \lambda^2 e^{3\lambda x} \]  
\[ \Gamma(1 + \alpha) \]  
(23)

\[ \Psi_2 = h^2t^{2\alpha} \frac{e^{5\lambda x} - \beta e^{-x} + \beta e^{2\lambda x} - x(\lambda - 1)\varepsilon \beta e^{\lambda x} - \lambda^2 e^{4\lambda x} + (3e^2 - 10)e^2 + 10\lambda e^{3\lambda x}}{\Gamma(1 + 2\alpha)} \]  
(24)
\[ \Psi_3 = i\hbar \alpha (1 + h)^2 e^{3\lambda x} + \beta e^{-x} \left(3 + 3h + h^2\right) + \varepsilon \lambda (1 + h)^2 e^{2\lambda x} - \lambda^2 (1 + h)^2 e^{\lambda x} + \frac{2(1 + h)e^{5\lambda x} - 10(1 + h)\varepsilon \lambda e^{4\lambda x} + (20 - 6\varepsilon^2)(1 + h)\lambda^2 e^{3\lambda x}}{\Gamma(1 + 2\alpha)} + \frac{12\varepsilon^3(1 + h)e^{2\lambda x} - 3\beta e^{2\lambda x} - 2h\beta e^{2\lambda x} - 2(1 + h)\lambda e^{\lambda x}}{\Gamma(1 + 2\alpha)} + \frac{(3 + 2h)e\beta (1 - \lambda)e^{\lambda x} + \beta (3 + 2h)e^{-x}}{\Gamma(1 + 2\alpha)} + \frac{3e^{7\lambda x} - 21\varepsilon \lambda e^{6\lambda x} + (55 - 34\varepsilon^2)e^{5\lambda x} + (138 - 12\varepsilon^2)e\lambda e^{4\lambda x} + \frac{4e^{-2} - 4\lambda + 3e^{-2} + 4\lambda - 2e^{-2}}{\Gamma(1 + 3\alpha)} + \frac{-2 + 4\lambda - 3\lambda^2 + \lambda^3}{\Gamma(1 + 3\alpha)} e\beta e^{\lambda x} + \beta e^{-x} + \frac{e^{7\lambda x} - \varepsilon \lambda e^{6\lambda x} + \beta e^{-2} - \left(2 + 4\varepsilon^2\right)\lambda^2 e^{5\lambda x}}{\Gamma(1 + \alpha)^2 \Gamma(1 + 3\alpha)} + \frac{2\left(\varepsilon \lambda^3 - \varepsilon \lambda^3 + \beta e^{-x}\right)e^{3\lambda x} + \left(1 - \lambda\right)e\beta e^{3\lambda x}}{\Gamma(1 + 3\alpha)} + \frac{3e^{2} + 1}{\Gamma(1 + \alpha)^2 \Gamma(1 + 3\alpha)} e\lambda e^{3\lambda x} + \left(e^2 - 2\lambda - 2e^2\lambda\right)e\beta e^{2\lambda x} - \frac{-\varepsilon \lambda^5 e^{2\lambda x} + \left(\beta e^{-x} - \varepsilon \lambda^2 + \varepsilon \lambda^3\right)\beta e^{\lambda x}}{\Gamma(1 + \alpha)^2 \Gamma(1 + 3\alpha)} \right) \] (25)

### 3.2. Solutions of ADM

In ADM [14], a nonlinear equation \( F[u(x,t)] = 0 \) is split as follows:

\[ F[u(x,t)] = L[u(x,t)] + R[u(x,t)] + N[u(x,t)] = 0 \] (26)

where \( L \) is a reversible linear operator, \( R \) is a linear irreversible operator, and \( N \) is the nonlinear part of the equation.

Using the inverse operator \( L^{-1} \) acting on the equation, we obtain

\[ u(x,t) = -L^{-1}R[u(x,t)] - L^{-1}N[u(x,t)] + \Phi \] (27)

Here \( \Phi \) satisfies \( L\Phi = 0 \)
We express the solution of the equation as a series

\[ u = \sum_{n=0}^{\infty} u_n \]  

(28)

Substitute the into the equation, we obtain

\[ \sum_{n=0}^{\infty} u_n = L^{-1} g - L^{-1} R \sum_{n=0}^{\infty} u_n - L^{-1} \sum_{n=0}^{\infty} A_n + \Phi \]  

(29)

That satisfy the following conditions

\[ u_0 = L^{-1} g + \Phi \]  

(30)

\[ u_1 = L^{-1} R u_0 + L^{-1} A_0 \]  

(31)

\[ u_2 = L^{-1} R u_1 + L^{-1} A_1 \]  

(32)

\[ \ldots \]

\[ u_n = L^{-1} R u_{n-1} + L^{-1} A_{n-1} \]  

(33)

To get the \( A_n \), we assume that the nonlinear part

\[ Nu = f(u) \]  

(34)

and

\[ u(x,t; \lambda) = \sum_{n=0}^{\infty} \lambda^n u_n (x,t; \lambda) \]  

(35)

Substitute the \( u(x,t; \lambda) = \sum_{n=0}^{\infty} \lambda^n u_n (x,t; \lambda) \) into equation, and assume that

\[ f(u(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n \]  

(36)

We expand \( f(u(\lambda)) \) at \( \lambda = 0 \), we obtain

\[ f(u(\lambda)) = f(u_0) + \left. \frac{df(u(\lambda))}{d\lambda} \right|_{\lambda=0} \lambda + \left. \frac{d^2 f(u(\lambda))}{d\lambda^2} \right|_{\lambda=0} \lambda^2 + \cdots + \left. \frac{d^n f(u(\lambda))}{d\lambda^n} \right|_{\lambda=0} \lambda^n + \cdots \]  

(37)

Compared with \( f(u(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n \), we obtain

\[ A_n = \frac{1}{n!} \left[ \frac{\partial^n}{\partial \lambda^n} N \left( \sum_{k=0}^{\infty} \lambda^k u_k \right) \right]_{\lambda=0} \]  

(38)

In this paper, we can choose the linear operator,

\[ L[\phi(x,t)] = D_t^\alpha [\phi(x,t)] \]  

(39)

the nonlinear operator,

\[ N[\phi(x,t)] = i\phi \phi + i\phi \frac{\partial \phi}{\partial x} + i \beta e^{-x} = \sum_{n=0}^{\infty} A_n \]  

(40)

and
\[ R[\phi(x,t)] = \frac{\partial^2 \phi}{\partial x^2} \] (41)

Operating with \( J^\alpha \) in both sides of Eq. (1), we obtain
\[ \Psi(x,t) = \Psi(x,0) + iJ^\alpha \left[ -\frac{\partial^2 \Psi(x,t)}{\partial x^2} + |\Psi(x,t)|^2 \Psi(x,t) + \varepsilon \Psi(x,t) \frac{\partial \Psi(x,t)}{\partial x} + i\beta e^{-x} \right] \] (42)

Then using the above formula, yield
\[ \Psi_0 = e^{\lambda x} \] (43)
\[ \Psi_1(x,t) = iJ^\alpha \left[ -\frac{\partial^2 \Psi_0(x,t)}{\partial x^2} + A_0 \right] \] (44)
\[ \Psi_2(x,t) = iJ^\alpha \left[ -\frac{\partial^2 \Psi_1(x,t)}{\partial x^2} + A_1 \right] \] (45)

\[ \cdots \]
\[ \Psi_n(x,t) = iJ^\alpha \left[ -\frac{\partial^2 \Psi_{n-1}(x,t)}{\partial x^2} + A_{n-1} \right] \] (46)

Where \( A_n \) are Adomian’s polynomials, which are derived as
\[ A_0 = \Psi_0 \overline{\Psi}_0 \Psi_0 + \varepsilon \Psi_0 \frac{\partial \Psi_0}{\partial x} + i\beta e^{-x} \] (47)
\[ A_1 = 2\Psi_0 \overline{\Psi}_0 \Psi_1 + \Psi_0^2 \overline{\Psi}_1 + \varepsilon \Psi_0 \frac{\partial \Psi_1}{\partial x} + \varepsilon \Psi_1 \frac{\partial \Psi_0}{\partial x} + i\beta e^{-x} \] (48)
\[ A_2 = 2\Psi_0 \overline{\Psi}_0 \Psi_2 + \Psi_0^2 \overline{\Psi}_2 + \Psi_1^2 \overline{\Psi}_0 + 2\Psi_1 \overline{\Psi}_1 \Psi_0 + \varepsilon \Psi_0 \frac{\partial \Psi_2}{\partial x} + \varepsilon \Psi_1 \frac{\partial \Psi_1}{\partial x} + \varepsilon \Psi_2 \frac{\partial \Psi_0}{\partial x} + i\beta e^{-x} \] (49)

\[ \cdots \]
\[ A_n = \sum_{k=0}^{n-1} \sum_{j=0}^{n-k-1} \Psi_k \overline{\Psi}_j \Psi_{n-k-j-1} + \varepsilon \sum_{k=0}^{n-1} \Psi_k \frac{\partial \Psi_{n-k-1}}{\partial x} + i\beta e^{-x} \] (50)

Where \( \overline{\Psi} \) is the conjugate of \( \Psi \).

From above, we obtain:
\[ \Psi_0 = e^{\lambda x} \] (51)
\[ \Psi_1 = i^\alpha \left[ e^{3\lambda x} - \beta e^{-x} - \varepsilon \lambda e^{2\lambda x} + \lambda^2 e^{\lambda x} \right] \frac{1}{\Gamma(1+\alpha)} \] (52)
\[ \Psi_2 = -ie^{-x} \frac{\beta}{\Gamma(1+\alpha)} - e^{5\lambda x} - 5\varepsilon \lambda e^{4\lambda x} + \beta e^{-x} + \frac{i\beta}{\Gamma(1+2\alpha)} \left[ 10 - 3e^x \right] e^{3\lambda x} + \frac{6\lambda^2 e^{2\lambda x} + (\lambda^4 - (1-\lambda) e^{\beta e^{-x}}) e^{\lambda x}}{\Gamma(1+2\alpha)} \] (53)
\[
\Psi_3 = -ie^{-x}t^\alpha + \frac{\beta}{\Gamma(1+\alpha)}e^{-x}t^\alpha + \frac{e^{2\lambda x} + (\lambda - 1)e^{\lambda x}}{\Gamma(1+2\alpha)} + \\
\frac{3\lambda^2 + 21\epsilon\lambda e^{6\lambda x} + 3\beta e^{-x} + (34\epsilon^2 - 55) e^{5\lambda x}}{\Gamma(1+3\alpha)} + \\
\frac{\left(12\epsilon^2 - 138\right)\epsilon\lambda^3 + 3\beta e^{-x} + \left(93 - 45\epsilon^2\right) + (6\lambda - 4)e^{-x}\right) e^{3\lambda x}}{\Gamma(1+3\alpha)} + \\
\frac{\left(-4 + \epsilon^2 + 4\lambda - 3\epsilon^2\lambda - 4\lambda^2 + 2\epsilon^2\lambda^2\right)e^{-x} + 26\epsilon\lambda^3}{\Gamma(1+3\alpha)} + \\
\frac{\left(-4 + \epsilon^2 + 4\lambda - 3\epsilon^2\lambda - 4\lambda^2 + 2\epsilon^2\lambda^2\right)e^{-x} + 26\epsilon\lambda^3}{\Gamma(1+3\alpha)} + \\
\frac{\frac{-e^{3\lambda x} + \epsilon\lambda e^{6\lambda x} - \beta^2 e e^{-2x} + (2 + 4\epsilon^2) e^{5\lambda x}}{\Gamma(1+3\alpha)} + \\
\frac{\left(2 - 4\lambda + 3\lambda^2 - \lambda^3\right)\epsilon\beta e^{-x} - \lambda^6}{\Gamma(1+3\alpha)} + \\
\frac{\left(\epsilon^2 - 1\right)\epsilon\lambda^3 - 2\beta e^{-x}}{\Gamma(1+3\alpha)} + \left(1 + 3\epsilon^2\right)\lambda^4 e^{3\lambda x}}{\Gamma(1+3\alpha)} + \\
\frac{\left(\lambda - 1\right)\epsilon\beta e^{3\lambda x - x} + \left(2\epsilon^2\lambda^2 + 2\epsilon^2 - \epsilon^2\lambda\right)e^{-x} + \epsilon\lambda^3}{\Gamma(1+3\alpha)} + \\
\frac{\epsilon\lambda^2 - \beta e^{-x} - \epsilon\lambda^3}{\Gamma(1+3\alpha)} e^{\lambda x} + \\
\frac{\epsilon\lambda^2 - \beta e^{-x} - \epsilon\lambda^3}{\Gamma(1+3\alpha)} e^{\lambda x}
\]

The above methodologies can also be used to find approximations to higher derivatives of a series solutions for both HAM and ADM. The derivation is algebraically tedious; therefore, only those solutions with special parameters are presented in the graphs.

4. RESULTS AND DISCUSSION

Liao [12] pointed out that the convergence of approximation for the HAM solutions strongly depend on the value of the auxiliary parameter \(\epsilon\). By means of the so-called \(\epsilon\)-curve, we can find a proper value of \(\epsilon\) which ensures that the solution series is convergent. In figure 1 and figure 2, we present the \(\epsilon\)-curves to show the convergent region. The imaginary and the real parts of the solution are plotted in the same figure. Figure 1 shows the \(\epsilon\)-curve of 6-th order approximate with HAM at \(x = 0.1, \ t = 0.2, \ \alpha = 0.8, \ \epsilon = 0.05, \ \beta = 0.1, \ \lambda = -1\), the convergent region is \([-1.2,-0.5]\). Figure 2 reveals the convergent region of the nonlinear equation with the \(\epsilon\)-curve of 6-th order approximate with HAM at \(x = 0.1, \ t = 0.2, \ \alpha = 0.9, \ \epsilon = 0.05, \ \beta = 0.1, \ \lambda = -1\), the convergent region is \([-1.1,-0.5]\). (In figure 1 and figure 2, the blue line represents the real part and the red line represents the imaginary part).
Figure 1. The h-curve of for 6-th order approximate with HAM at $x = 0.1$, $t = 0.2$, $\alpha = 0.8$, $\epsilon = 0.05$, $\beta = 0.1$, $\lambda = -1$.

Figure 2. The h-curve of for 6-th order approximate with HAM at $x = 0.1$, $t = 0.2$, $\alpha = 0.9$, $\epsilon = 0.05$, $\beta = 0.1$, $\lambda = -1$.

Figure 3 presents a comparison of the approximate solution $|\Psi|^2$ between ADM and HAM ($\epsilon = 0.05$, $\alpha = 0.9$, $\beta = 0.1$, $\lambda = -1$). It is seen that the results obtained by the two methods are in very close agreement.
Figure 3 The comparison of the 6-th order approximate solution $|\Psi|^2$ between ADM and HAM at $\varepsilon = 0.05$, $\alpha = 0.8$, $\beta = 0.15$, $\lambda = -1$.

Figure 4 indicates the influence of the fractional parameter $\alpha$ on the probability density profiles. It is seen that for special (finite) values of $\varepsilon$ and $\beta$, the fractional parameter $\alpha$ has a significant influence on the probability density profiles.

Figure 4 The 6-th order approximate solution $|\Psi|^2$ with HAM at $\alpha = 1.0$, $\alpha = 0.95$, $\alpha = 0.9$, $\varepsilon = 0$, $\beta = 0$, $\lambda = -1$. 
Figure 5 The comparison of the 6-th order approximate solution $|\Psi|^2$ with HAM

at $\varepsilon=-0.05$, $\varepsilon=0$ and $\varepsilon=0.05$ at $\alpha=0.8$, $\beta=0$, $\lambda=-1$

Figure 5 presents the influence of the small disturbance parameter $\varepsilon$ on the probability density function. The disturbance is about the potential energy. The figure indicates that the probability density $|\Psi|^2$ monotonic increases with the increasing in parameter $\varepsilon$. It implies that the probability density function is not only a function of the location $x$, but also related to the wave function and the derivative of wave function respect to $x$.

Figure 6 describes the influence of the small disturbance parameter $\beta$ on the probability density profiles. It is seen that the probability density profile decreases with the increasing $\beta$. The influence of the parameter $\beta$ on the probability density is opposite to the influence of the parameter $\varepsilon$.

Figure 6 The comparison of the 6-th order approximate solution $|\Psi|^2$ with HAM

at $\beta=-0.15$, $\beta=0$ and $\beta=0.15$ at $\alpha=0.8$, $\beta=0$, $\lambda=-1$

Figure 7 shows the influence of the parameter $\lambda$ in the initial condition on the probability density. The figure indicates that the initial value parameter $\lambda$ has also a significant influence on the probability density profiles. The value of the probability density increases dramatically as the decreasing in the initial value parameter $\lambda$. 

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