A MATHEMATICAL MODELING APPROACH TO STUDY GROWTH RATE OF GRASSROOTS TECHNOLOGICAL INNOVATIONS

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ABSTRACT

In this paper we have proposed a simple mathematical model by using ordinary differential equation to know the spread rate of technological innovations in rural India. In support of model the data related to technological innovations in the state Punjab (India) has been collected and analysed graphically/mathematically. The outcomes verified the Sigmoid pattern growth (exponential, linear & asymptotic)

Keywords: *Ordinary differential equations, technological innovations, mathematical modeling*

1. INTRODUCTION

The development of any country is a dynamic phenomenon and has to be constantly endeavored. Ever since India became a sovereign republic, the successive Governments in all the States have been endeavoring for the rapid economic development. Here in the present study we have studied the technological innovation (TI) of one of the most fertile and prosperous Indian state, Punjab as a case study. In early sixties the establishment of an Agricultural University in Punjab paid rich dividends in the shape of green revolution and white revolution. As a result, agriculture sector got a big boost. The green revolution, the white revolution and blue revolutions helped continued economic growth of the State, in spite of a decade of turmoil in the recent past.

Once an innovation has been introduced by one industry/firm/or a group of people/or by an individual, how soon do others adopt it in rural India? What are important factors which determine how rapidly the innovation spreads, what is the trend to spread rate of TI, these are the important questions for sociologists, economists, scientists, managers, technocrats & ultimately governments! Keeping this in mind, in this paper we have studied "A Mathematical modeling approach to study growth rate of grassroots Technological innovations" for prediction of adoption rate of new innovations among Indian villagers especially who are depending on agriculture only. For the purpose we have discussed an application of ordinary differential equation to construct a mathematical model to know the spread of technological innovations in rural India. Here we have predicted the adoption process of TI as sigmoid pattern growth or almost sigmoid pattern. In support of this, we have taken Punjab (India) as a case study and for the purpose; we have collected the data regarding technological innovations in farming about:

a) Percentage of area in thousand hectares under high yielding varieties of major food Crops.
b) Consumption of Chemical Fertilizers in Punjab
c) Consumption of Pesticides/Insecticides in Punjab
d) Tractors registered in Punjab
e) Number of Tube wells in Punjab

2. MATHEMATICAL MODELING

Let us suppose that the innovation is introduced in to a group of n farmers/villagers, who have adopted the innovation at time t [1]. Obviously $x(t)$ has integer values but we approximate to it as continuous functions of time.

Now a villager/farmer will generally adopt the innovation only after a farmer who already uses it has told him about it.

Therefore we assumed that the number of farmer /villagers are δx, who adopt the innovation in a small time interval δt.

Hence

i) $\delta x \propto$ the number of farmers x (t) who have already adopted.

ii) $\delta x \propto$ the number of farmers $\{n - x(t)\}$ who have not adopted.

Thus we can write

$$\delta x = c_1 x(n-x)\delta t \text{ ,} \quad \text{where } c_1 \text{ is the Positive constant}$$

Now we can write

$$\frac{dx}{dt} = c_1 x(n-x)$$

On solving, we have

$$\log\left|\frac{x}{n-x}\right| = nc_1 t + A$$

Where A is integration constant which can be calculated after taking initial conditions

Initial conditions

If at $t = 0; n = n_0 \;\&\; x = x_0$ then we can write

$$A = \log\left[\frac{x_0}{n_0 - x_0}\right]$$

Now putting this value in the above equation we have

$$\log\left[\frac{x}{n-x}\right] = nc_1 t + \log\left[\frac{x_0}{n_0 - x_0}\right]$$

But if $x_0 = 1$, After simplification we have

$$x[n_0 - 1 + e^{nmt}] = ne^{nc_1 t}$$

or $$x = \left[\frac{ne^{nc_1 t}}{n_0 - 1 + e^{nc_1 t}}\right]$$

Interpretation of the model:

Assuming that $x(0) = 1$, this gives the shape like 'verhuest Population model' [3]

Which is familiar as *SIGMOID SHAPE MODEL* on analyzing mathematically the above equation it can be predicted that adoption process increase up to the value n/2 means, when 50% of the farmers/villagers have adopted the innovation after which the adoption process slows down. One source of any discrepancy might be the assumption that a farmer only learns of an innovation from another farmer or villager. He/She could easily hear of it through advertising and this could well play a significant part in the adoption process, particularly in its early stages.

Let us suppose that in the small time δt , the numbers of farmers or villagers being influenced through the mass media, which is proportional to the number of farmers or villagers who have not adopted the innovation

i.e. $c_2(n-x)\delta t$, where c_2 is a positive constant.

Now the governing equation will be

$$\delta x = c_1 x(n-x)\delta t + c_2(n-x)\delta t$$

This gives the differential equation

$$\frac{dx}{dt} = (c_1 x + c_2)(n-x)$$

Which is a variable-separable equation which can be solve easily

$$\int \frac{dx}{(x+c)(n-x)} = \int c_1 dt$$

Where $c = \frac{c_2}{c_1}$ Thus

After integration we get

$$\frac{x+c}{n-x} = be^{(n+c)c_1 t}$$

Solving for x and with the initial condition $x(0) = 1$, gives

$$x = \frac{(c_1 + c_2)ne^{(c_1 n + c_2)t} - c_2(n-1)}{(n-1)c_1 + (c_1 + c_2)e^{(c_1 n + c_2)t}}$$

We again have *SIGMOID SHAPE TYPE GROWTH.*

Assuming that $x(0) = 1$, which gives the sigmoid shape and so predicts that the adoptions process increases up to the values n/2. [2] & [4]

3. VERIFICATION OF MATHEMATICAL MODEL

Now we test the predictions of model with some actual data on Technological innovations in farming (about Indian state Punjab as a case study)

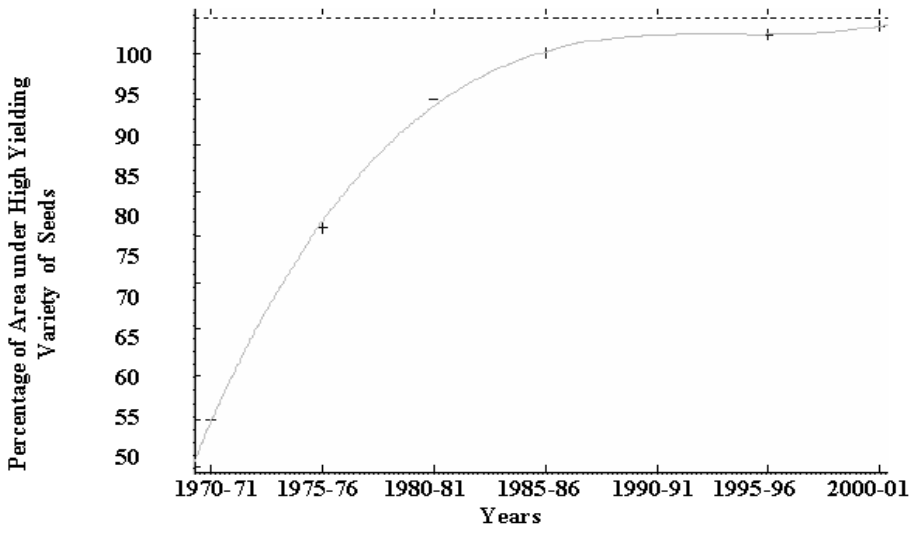
The high rate of food grains production in Punjab has been accompanied by rapid adoption of Technological Innovations such as high yielding varieties (HYVs) of seeds, Chemical Fertilizers, Pesticides/Insecticides, Tube wells and Tractors. With the help of data, we have studied in details about technological innovations at grass root level.

a) Percentage of area in thousand hectares under high yielding varieties of seeds of major food crops (Rice, Maize, Millet & Wheat) to the total area sown under these crops in Punjab [6].

Years	**1970-1971**	**75-76**	**80-81**	**85-86**	**90-91**	**95-96**	**2000-01**
Percentage of area under High yielding varieties of crops (In %)	55	76	90	95	97	97	98

(Table No:1)

Graph as per above data:



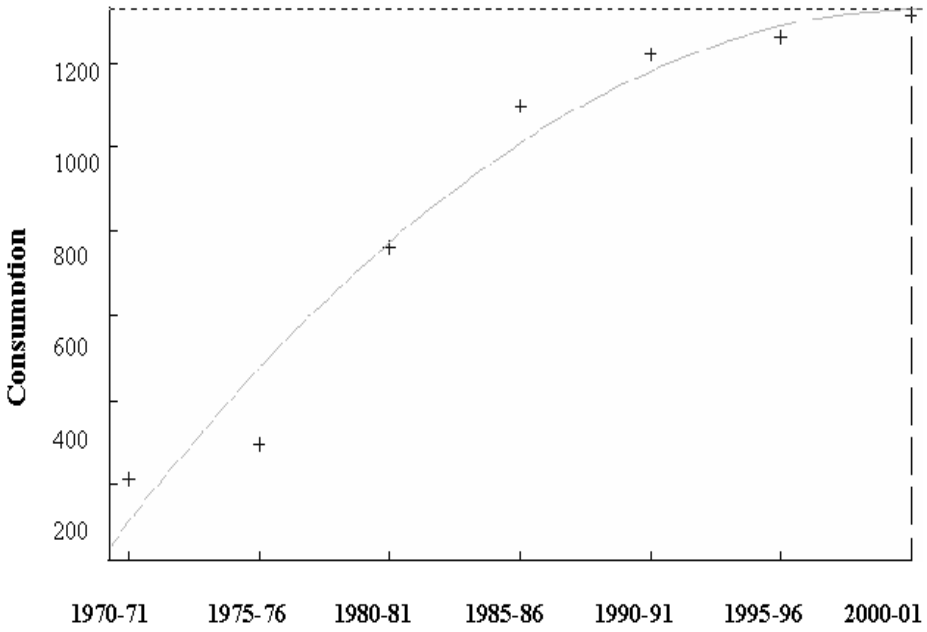
(Fig: 1)

b)Consumption of Chemical Fertilizer in Punjab (In 000' Nutrient tones) [5].

Years	**1970-1971**	**75-76**	**80-81**	**85-86**	**90-91**	**95-96**	**2000-01**
Consumptions	213	295	762	1098	1220	1263	1313

(Table No :2)

Graph as per above data:



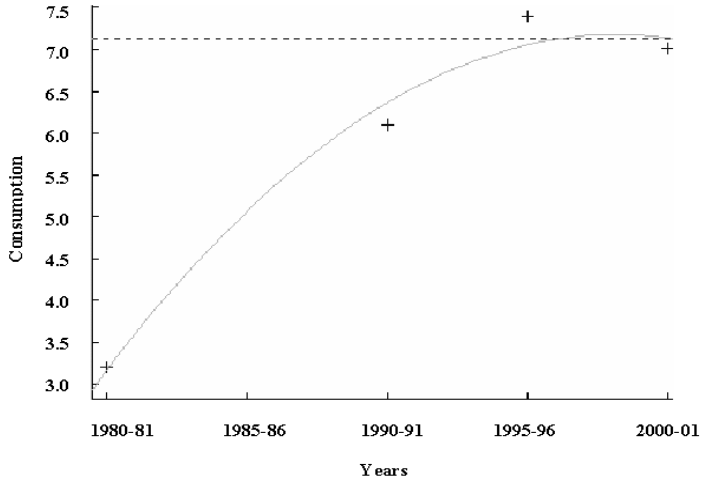
(Fig.-2)

c) Consumption of Pesticides/Insecticides in Punjab (In 000' metric tones) [6].

Years	**1980-1981**	**1989-1990**	**1994-1995**	**1999-2000**
Consumptions	3.2	6.0	7.3	6.9

(Table NO: 3)

Graph as per above data:



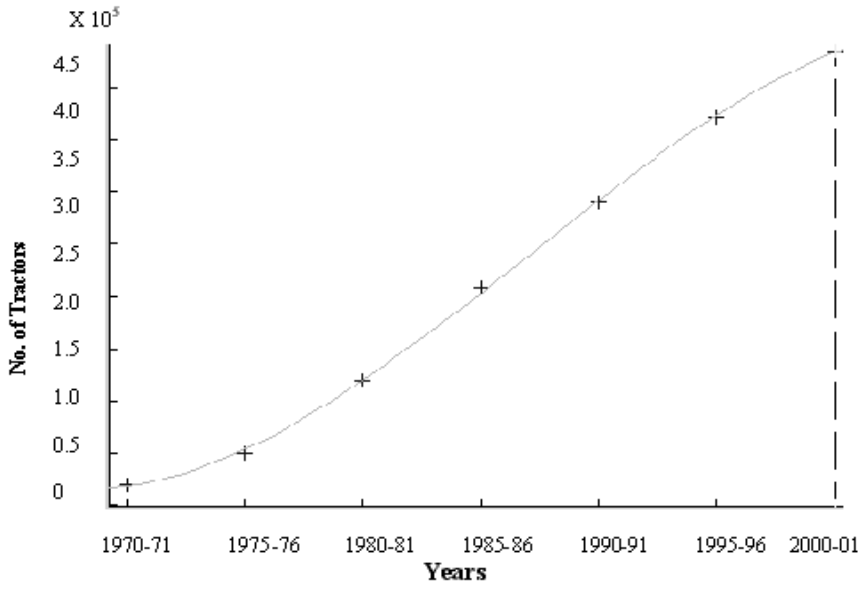
(Fig.-3)

d) Tractors registered in Punjab [5].

Years	**1970-1971**	**1975-1976**	**1980-1981**	**1985-1986**	**1990-1991**	**1995-1996**	**2000-2001**
Number of Tractors	20519	50664	118845	208614	289064	371720	434032

(Table No: 4)

Graph as per above data:



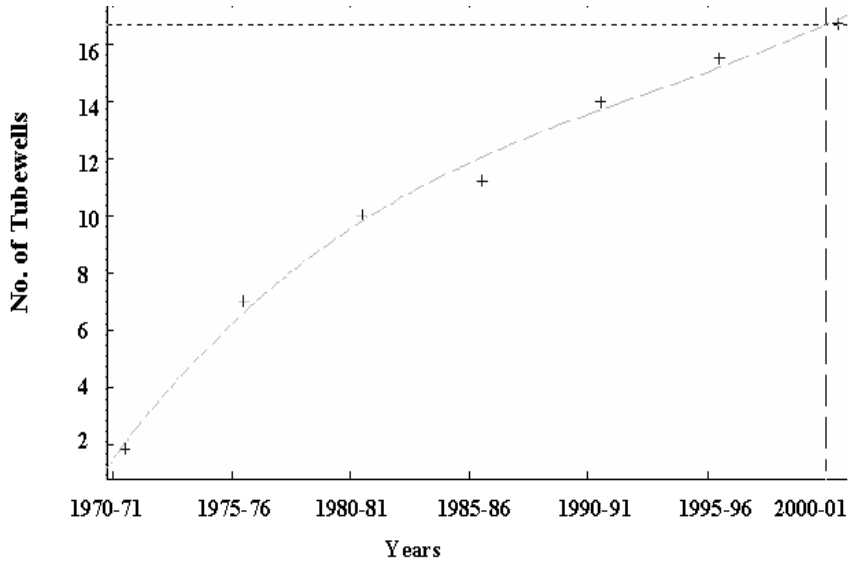
(Fig.-4)

e) Number of Tube wells in Punjab (In lacs)[5]

Years	**1970-1971**	**1975-1976**	**1980-1981**	**1985-1986**	**1990-1991**	**1995-1996**	**2000-2001**
Number of Tubewells	1.92	4.5	6.0	6.62	8.0	8.75	9.35

(Table No: 5)

Graph as per above data:



(Fig- 5)

All these curves follow sigmoid type shapes and so offer some support to proposed Mathematical model.

4. OBSERVATIONS FROM THE GRAPHS

Following is the Observations from the above-discussed Graphs:

- In Fig (1) of Percentage of area under High yielding variety of seeds in major food crops, we observed that adoption curve was exponential between 1970 to 1980. After that growth was not so rapid. It became linear from 1980 to 1985; afterward it seems to be saturated to cent-percent line as farmers in Punjab adopted nearly 100% high yielding varieties of seeds in major food crops.
- In Fig (2) we have observed that curve of consumption of chemical fertilizers in Punjab during 1970 to1985 was exponential. After that growth of innovation was not so rapid. It became approximately linear between 1985 to 1990 and after 1990 it seems to become saturated to the line of above 1200 metric nutrient tones.
- In Fig (3) we have observed that the curve of consumption of chemical fertilizers in Punjab is exponential from 1980 to 1990, it became linear from 1990 to 1995. After that growth become saturated to the line of 7000 metric tones. Approximately saturation level attained.
- In Fig (4) for the registration of tractors in Punjab, curve was exponential from 1970 to 1990. Afterward, growth rate of innovation didn't look rapid. It became linear after 1990.and towards saturation; this curve is much closer to sigmoid curve.

- In Fig. (5) curve for number of tube wells in Punjab is exponential from 1970 To 1990. After that the growth rate of installing new tube wells was not so rapid. Curve became linear after 1990.and then towards saturation.

5. CONCLUDING REMARKS

On analyzing mathematically, it may be predicted that adoption process increases up to the value n/2 means, when 50% of the farmers/villagers have adapted the innovation after which the adoption process slows down, it is also very much clear that The most common pattern seen under relatively stable conditions is a "sigmoid" pattern where this variable sequentially goes through phases that appear to be exponential, then linear, and finally asymptotic to some upper limit in all the cases of innovations in farming. One source of any discrepancy might be the assumption that a farmer only learns of an innovation from another farmer or villager. He/She could easily hear of it through advertising and this could well play a significant part in the adoption process, particularly in its early stages. During the study as discussed in the paper again we got the sigmoid pattern and data's are best fitted in the proposed model. As it is quite clear from the above discussed graphs in support of technological innovations in villages as high yielding varieties of major food crops (Table No. 1), Consumption of Chemical Fertilizer in Punjab (Table No. 2), Consumption of Pesticides/Insecticides in Punjab (Table No. 3), Tractors registered in Punjab (Table No. 4) & Number of Tube wells in Punjab (Table No. 5). All the different curves have similar shapes these curves resembles like S and so it is called an S-shaped curve or a sigmoid pattern growth. As anyone can easily see, when the innovations starts to grow, it does go through an exponential growth phase, but as it gets closer to the carrying capacity, the growth slows down and it reaches almost a stable level. Which support the Sigmoid Pattern of the proposed Mathematical model. To expedite the spread of technological innovations, role of Information Technology and Media must be encouraged. The widespread availability and convergence of information and communication technologies (ICTs) like computers, digital networks, telecommunication, television etc in recent years have led to unprecedented capacity for dissemination of knowledge and information to the rural population around the world. It can play a very important role to spread the technological innovations globally in more rapid form.

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