

# RELIABILITY CHARACTERISTIC OF COLD-STANDBY REDUNDANT SYSTEM

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## ABSTRACT

The objective of this paper is to improve the system reliability through append the redundant component. In this paper, two units in cold-standby is considered. Each unit of the system has two modes via; operable and failed. The failures of units are of two types: minor and major. After major failure, cold standby unit replaces the failed unit after a random amount of time. The failure and repair times follow exponential and general time distributions respectively. We transform the basic equations of the proposed model into an integro-differential equation and solve it using Supplementary variables techniques, various reliability parameters have been computed and analyzed by tabular and graphical illustrations.

**Keywords:** *Steady-state behaviour, Redundancy, Cold-standby*

## 1. INTRODUCTION

Redundancy plays an important role in enhancing system reliability. The redundancy allocation problem has been analyzed for many different system structure [6 ,8,11]. One of the commonly used form of the redundancy is the standby redundancy. Standby system often finds applications in various industrial and other setups. In a standby redundant system, some additional paths are created for the proper functioning of the system. Standby unit is support to increase the reliability of the system.

In general there are 3-types of standby i.e. cold, hot and warm standby. Cold standby means that the redundant components cannot fail while they are waiting. Earlier many researchers [1, 3, 12] have discussed when an operative unit fails, its repair starts immediately but in practical problem it may not possible always. In practice, system do not always fail with major breakdown. Igichart and Igichart and lemoline; Arti R.(1993); Singh, S.B.(1998) developed various mathematical models consists two types of failure namely major and minor.

A maintenance policy that suits a system presenting two types of failure represented by many earlier researchers [2,10]. Revealed minor failures are removed by a minor repair that brings the system back to the operating condition just previous to failure. A major repair is restore the system as good as new. Keeping this fact in view, the author has considered a complex system consisting of an operative unit and cold-standby unit having two repair facilities. Initially the system starts functioning in a normal efficiency state. When the operative unit fails, the standby units starts functioning. Due to imperfect switching system stops working and goes to failed state  $P_A$  then the standby unit starts functioning and goes to the state  $P_B$ , further if there is major failure in A, system goes to the state  $P_{W_A}$ . moreover if the unit B breakdown then system goes to failed state  $P_F$  and it will repair. Using Supplementary variables technique Laplace transforms of various state probabilities have been evaluated. Numerical examples have also been added to highlight the important results.

## 2. ASSUMPTIONS

- 1) Initially system is in operable state.
- 2) The system consists of two unit one operative and other cold-standby.
- 3) If both units fail, the whole system fails completely.
- 4) The repair facility is not always available with the system, but is called upon whenever the operative units fails
- 5) The system has two repair facilities.
- 6) Failure and repair time follow exponential and general time distribution respectively.

3. NOTATIONS

- $\lambda_A, \lambda_B$  = constant failure rate of units A and B
- $\mu_A(x), \mu_{A_M}$  =Minor and Major repair rate of unit A
- $\mu(x)$  =Repair rate of both units A and B
- $W_S$  =Time for inspecting the switch
- $W_A$  =weighting time for calling the repairman for unit A
- $P_0(t)$  =Probability of the system in operable state
- $P_F(t)$  =Probability of the system in failed state
- $P_A(t)$  =Probability of the unit A in failed state
- $P_B(t)$  =Probability of the unit B In operable state
- $P_{W_A}(t)$  =Probability of weighting time

State Transition Diagram

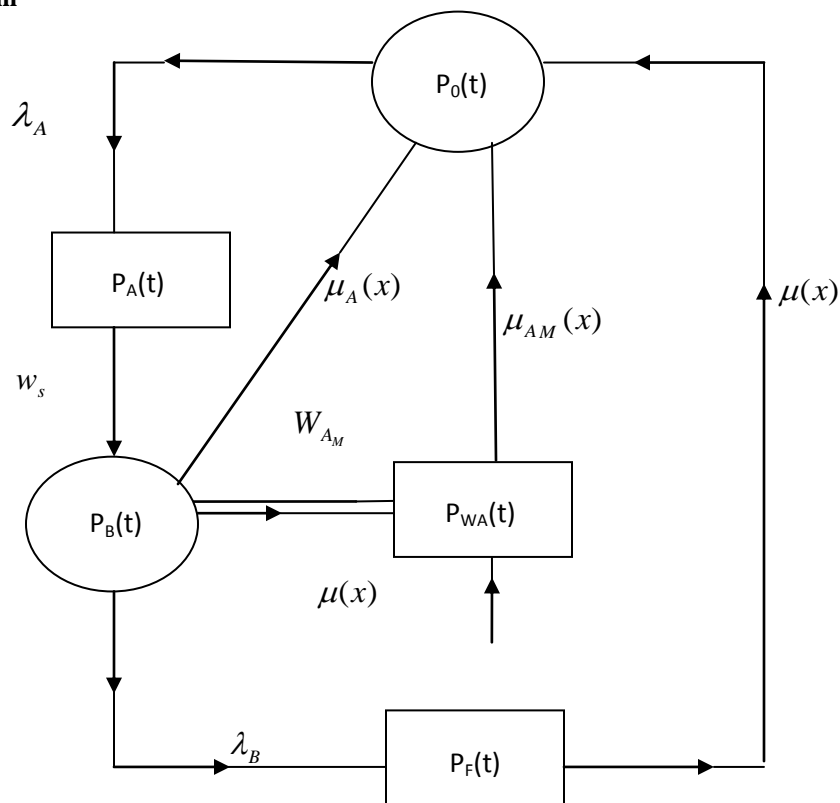


Fig. 1

4. FORMULATION OF THE MATHEMATICAL MODEL

By elementary and continuity arguments, the difference-differential equations governing the stochastic Behaviour of the complex system are:

$$\left[ \frac{\partial}{\partial t} + \lambda_A \right] P_0(t) = \int_0^{\infty} \mu_A(x) P_B(x,t) dx + \int_0^{\infty} \mu_{A_m}(x) P_{W_A}(x,t) dx + \int_0^{\infty} \mu_A(x) P_f(x,t) dx \quad \dots(1)$$

$$\left[ \frac{\partial}{\partial t} + w_s \right] P_A(t) = \lambda_A P_0(t) \quad \dots(2)$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_A(x) + w_A + \lambda_B \right] P_B(x,t) = 0 \quad \dots(3)$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_{A_m}(x) \right] P_{W_A}(x,t) = 0 \quad \dots(4)$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\mu(x) \right] P_f(x,t) = 0 \quad \dots(5)$$

### BOUNDARY CONDITIONS

$$P_B(0,t) = W_s P_A(t) + \int_0^{\infty} \mu(x) P_f(x,t) dx \quad \dots(6)$$

$$P_{W_A}(0,t) = W_A P_B(t) \quad \dots(7)$$

$$P_f(0,t) = \lambda_B P_B(t) \quad \dots(8)$$

### INITIAL CONDITIONS

$$P_0(t) = 1, \text{ otherwise zero} \quad \dots(9)$$

### 5. SOLUTION OF THE MODEL

Taking Laplace transform of (1) to (9) and on further simplification, one may obtain

$$\bar{P}_0(s) = \frac{1}{A(s)} \quad \dots(10)$$

$$\bar{P}_A(s) = \frac{\lambda_A}{(s + W_s)} \cdot \frac{1}{A(s)} \quad \dots(11)$$

$$\bar{P}_B(s) = \frac{W_s \lambda_A}{(s + W_s)} \left\{ \frac{1 - \bar{S}_A(s + W_A + \lambda_B)}{(s + W_A + \lambda_B)} \right\} \cdot \frac{1}{B(s)} \cdot \frac{1}{A(s)} \quad \dots(12)$$

$$\bar{P}_{W_A}(s) = \left[ \frac{W_A W_s \lambda_A}{(s + W_s)} \left\{ \frac{1 - \bar{S}_A(s + W_A + \lambda_B)}{(s + W_A + \lambda_B)} \right\} \left\{ \frac{1 - \bar{S}_{\mu_{A_m}}(s)}{s} \right\} \frac{1}{B(s)} \right] \frac{1}{A(s)} \quad \dots(13)$$

$$\bar{P}_f(s) = \left[ \frac{W_s \lambda_A \lambda_B}{(s + W_s)} \left\{ \frac{1 - \bar{S}_A(s + W_A + \lambda_B)}{(s + W_A + \lambda_B)} \right\} \left\{ \frac{1 - \bar{S}_{2\mu}(s)}{s} \right\} \frac{1}{B(s)} \right] \frac{1}{A(s)} \quad \dots(14)$$

Where,

$$A(s) = s + \lambda_A - \frac{W_s \lambda_A \bar{S}_A(s + W_A + \lambda_B)}{(s + W_s)} - \left[ \left\{ \frac{W_s \lambda_A}{(s + W_s)} \cdot \frac{1 - \bar{S}_A(s + W_A + \lambda_B)}{(s + W_A + \lambda_B)} \frac{1}{B(s)} \right\} \left\{ W_A \bar{S}_{\mu_{Am}}(s) + \frac{\lambda_B}{2} \bar{S}_{2\mu}(s) + \frac{\lambda_B}{2} \bar{S}_{2\mu}(s) \bar{S}_A(s + W_A + \lambda_B) \right\} \right] \quad \dots (15)$$

It is worth nothing that

$$\bar{P}_0(s) + \bar{P}_A(s) + \bar{P}_B(s) + \bar{P}_{W_A}(s) + \bar{P}_f(s) = \frac{1}{s}$$

### 6. EVALUATION OF L.T. OF UP AND DOWN STATE PROBABILITIES

The L.T.of the probabilities that the system is in operable and down state at time ‘t’ can be evaluated as follows:

$$\begin{aligned} \bar{P}_{UP}(s) &= \bar{P}_0(s) + \bar{P}_B(s) \\ &= \left[ 1 + \frac{W_s \lambda_A}{(s + W_s)} \left\{ \frac{1 - \bar{S}_A(s + W_A + \lambda_B)}{(s + W_A + \lambda_B)} \right\} \frac{1}{B(s)} \right] \frac{1}{A(s)} \quad \dots(16) \end{aligned}$$

$$\begin{aligned} \bar{P}_{down}(s) &= \bar{P}_A(s) + \bar{P}_{W_A}(s) + \bar{P}_f(s) \\ &= \left[ \frac{\lambda_A}{(s + W_s)} + \frac{W_s \lambda_A}{(s + W_s)} \left\{ \frac{1 - \bar{S}_A(s + W_A + \lambda_B)}{(s + W_A + \lambda_B)} \right\} \left\{ W_A \left( \frac{1 - \bar{S}_{\mu_{Am}}(s)}{s} \right) + \lambda_B \left( \frac{1 - \bar{S}_{2\mu}(s)}{s} \right) \right\} \frac{1}{B(s)} \right] \frac{1}{A(s)} \quad \dots(17) \end{aligned}$$

### 7. ERGODIC BEHAVIOUR

Using Abel’s Lemma

$$\lim_{s \rightarrow 0} s \bar{f}(s) = \lim_{t \rightarrow \infty} f(t) = f(say)$$

Provided that the limit on the right hand side exists, the time independent up and down state probabilities are obtained ass follows:

$$P_{UP} = \left[ 1 + \frac{\lambda_A}{(W_A + \lambda_B + \mu_A)B(0)} \right] \frac{1}{A'(0)} \tag{18}$$

$$P_{DOWN} = \left[ \frac{\lambda_A}{W_s} + \frac{\lambda_A}{(W_A + \lambda_B + \mu_A)} \left\{ \frac{W_A}{\mu_{A_m}} + \frac{\lambda_B}{2\mu} \right\} \frac{1}{B(0)} \right] \frac{1}{A'(0)} \tag{19}$$

**8. RELIABILITY**

The reliability of the system is given by

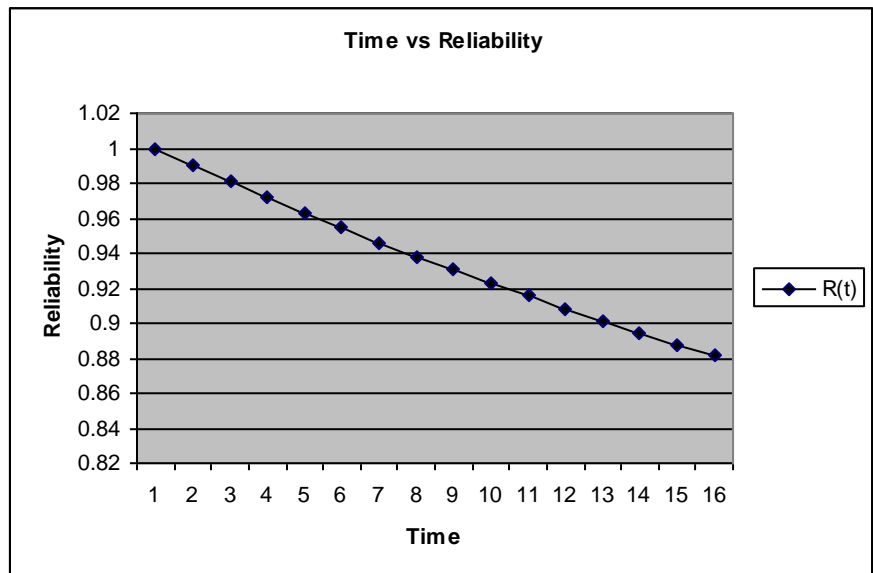
$$R(s) = \frac{1}{A(s)} \left[ 1 + \frac{w_s \lambda_A}{(s + w_s)(s + \lambda_B + w_A)} \right]$$

Where  $A(s) = s + \lambda_A$

Assuming  $\lambda_A = 0.01, \lambda_B = 0.02, w_s = 0.03, w_A = 0.04$ , one may obtain

$$R(t) = 1.3 e^{-0.01t} + 0.2 e^{-0.06t} - 0.5 e^{-0.03t}$$

Time	R(t)
0	1
1	0.990195
2	0.98076
3	0.971668
4	0.962892
5	0.954408
6	0.946194
7	0.938229
8	0.930494
9	0.92297
10	0.915462
11	0.908493
12	0.901509
13	0.894677
14	0.887984



**Fig:2**

**9. MEAN TIME TO FAILURE**

The mean time to failure (MTTF) of the system is given by

$$MTTF = \frac{\lambda_B + w_A + \lambda_A}{(\lambda_B + w_A)\lambda_A}$$

$\lambda_A$	MTTF $w_A = 0.015$	MTTF $w_A = 0.035$	MTTF $w_A = 0.055$
0.001	0.001025	0.001017	0.001013
0.002	0.0021	0.002067	0.00205
0.003	0.003225	0.00315	0.003113
0.004	0.0044	0.004267	0.0042
0.005	0.005625	0.005417	0.005313
0.006	0.0069	0.0066	0.00645
0.007	0.008225	0.007817	0.007613
0.008	0.0096	0.009067	0.0088
0.009	0.011025	0.01035	0.010013
0.01	0.0125	0.011667	0.01125

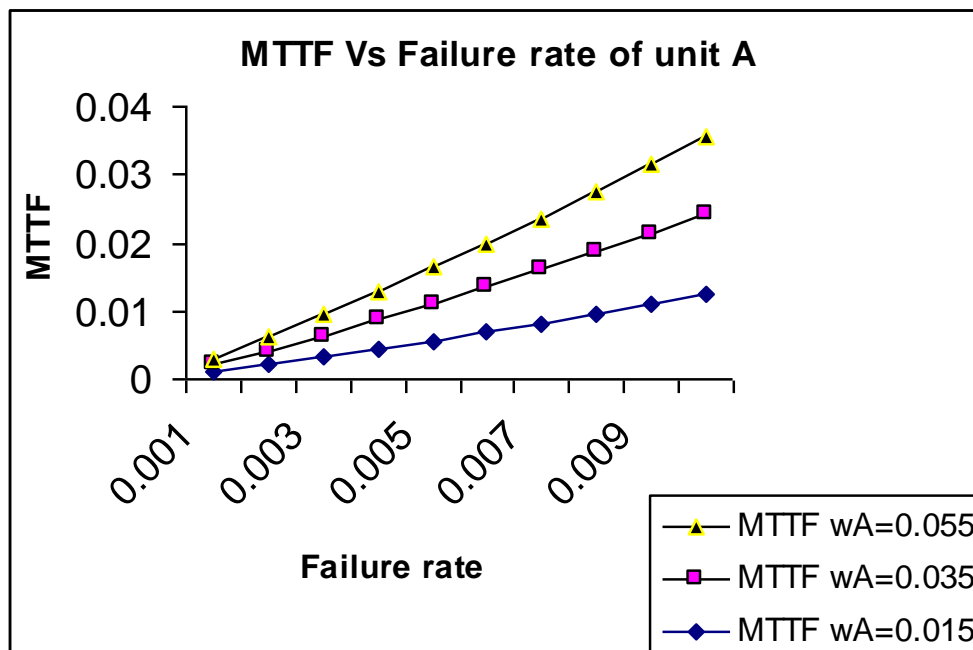


Fig:3

## CONCLUSION

From fig: 2 it is clear that the Reliability of the system decreases as the time period increases and fig: 3 display the variation in MTTF as the minor failure of unit A increases. The series of curve represent that MTTF increases as the failure rate increases.

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