

EVALUATION OF THE RISK OF DRUG ADDICTION WITH THE HELP OF FUZZY SETS

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ABSTRACT

The primary focus of this paper is to present a general view of the current applications of fuzzy logic in medical analogy of consumption of drugs. The paper also deals with the origin, structure and composition of fuzzy sets. We particularly review the medical literature using fuzzy logic. Fuzzy set theory can be considered as a suitable formalism to deal with the imprecision intrinsic to many real world problems. Fuzzy set theory provides an appropriate framework for the representation of vague medical concepts and imprecise modes of reasoning. We present two concrete illustrations to investigate the impact of the risk related to drug addictions, like smoking and alcohol drinking and thereby highlighting the social problem related to health.

Keywords: *Estimation, Classical set, Validation, Diagnostic test, Linguistic variable, Risk assessment, Crisp set.*

1. INTRODUCTION

Logic studies the notions of consequence it deals with propositions, set of propositions and the relation of consequence among them. Formal logic represents this by means of well-defined logical calculi. Logical calculus has two notions of consequence they are syntactical, based on a notion of proof and semantical which are based on notion of truth. Fuzzy propositions use linguistic variables such as AGE; with values -young, very old, old. The truth of a fuzzy proposition is a matter of degree. Aristotle in his, laws of thought and law of the excluded middle described every proposition must either be True or False. Plato laid the foundation, for what would become fuzzy logic, indicating that there was a third region beyond True and False. Lukasiewicz who first proposed a systematic alternative to the bi-valued logic of Aristotle. He described the 3-valued logic, with the third value is Possible. He showed that it is possible to derive infinite-valued logic. In 1965 Lotfi Zadeh published his seminal work on "Fuzzy Sets" described mathematics of fuzzy set theory. In 1973 Lotfi Zadeh proposed his theory of fuzzy logic and there by the membership function operates over the range of real numbers [0, 1]. New operations for the calculus of logic were proposed, and shown to be in principle at least a generalisation of classic logic. Lotfi Zadeh is regarded as the father of fuzzy theory. He was born in Iran and graduated from the University of Tehran. In the United States he received his doctoral degree at Columbia University. He worked at Princeton university and then became a professor at the university of California in 1959. He was brilliant researcher in control theory and system theory before he proposed a theory whose objects that is, fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree.

The word Fuzzy means indistinct, imprecise, obscure, blurred, vague, ambiguous etc. Hence one may be tempted to interpret fuzzy set as a vague set or ambiguous set but this would be a wrong interpretation. In fact, 'fuzzy set' is a well defined concept in mathematics. The driving force behind this change is the realisation that classical set theory is inadequate for dealing with imprecision, uncertainty and the complexity of the real world. This motivates the evolution of fuzzy set theory. The example AGE with different values young, very old, old, containing adjective, verb and adverb may not form sets in the usual mathematical sense, the fact remains that such imprecisely defined classes play an important role in human thinking, particularly in the domain of pattern recognition, digital communication and Information technology. Zadeh describes fuzzy logic as formalisation of approximate reasoning. Although "fuzzy logic" may seem to imply imprecision, it's based on a reliable and rigorous discipline. Fuzzy logic lets us accurately describe control systems in words instead of complicated math. Fuzzy logic, based on fuzzy set theory, allows us to express the operational and control laws of a system linguistically in words. Although such an approach might seem inadequate, it can actually be superior to and much easier than a more mathematical approach. The main strength of fuzzy set theory, a generalization of classical set theory, that excels in dealing with imprecision. In classical set theory, an item is either a part of a set or not. There is no in-between, there are no partial members. Fuzzy set theory recognizes that very few crisp sets actually exist [1]. There is a paradigm shift from crisp

set to fuzzy set. The paradigm shift is necessitated by “the need to bridge the gap between mathematical models (biology, medicine, social science) and experience”. This paradigm allows to express observation and measurement uncertainties, manage complexity, and capture human common-sense reasoning and decision making.

2. NON FUZZY (CLASSICAL) SETS TO FUZZY SETS

One of the important tools in modern mathematics is the theory of sets. Every branch of mathematics can be considered as a study of sets of objects of one kind or another. For instance geometry is the study of set of points. Algebra is concerned with the set of numbers and operations on those sets [7]. Analysis mainly deals with set of function. The study of sets and their use in the foundation of mathematics began in the latter part of the nineteenth century by German mathematician George Cantor (1845-1918). According to Cantor, “A set is a collection into a whole of definite and distinct objects of our intuition or thought and the objects are called ‘elements’ of the set”. About the turn of the twentieth century paradoxes of various kinds namely Russell’s paradox (1901), Cantor’s paradox (1932), Burali-Forti’s Paradox (1897) were discovered which directly or indirectly originated from the notion of the set and which shook the foundations of mathematics in general and set theory in particular. Frege, indeed admitted that Russell’s paradox undermines the foundations of his life work to construct arithmetic on the basis of the set theory. There are three principal philosophies of mathematics each of which has a sizable group of adherents. These are logistic school, of which Frege, Russell and White- Head are the main expositors, the intuitionist school was led by the Dutch mathematician L.E.J. Brouwer and Heyting and the formalist school developed principally by David Hilbert. These Schools of mathematics were greatly influenced by the appearance of paradoxes. The mathematicians of these schools approached the problem posed by the paradoxes according to the philosophy of mathematics they held. The basic assumption that has been made in any axiomatic set theory as well as in Cantor’s intuitive set theory is that given any set A and any object x of the universe of discourse X it can be decided whether $x \in A$ holds or not. Hence corresponding to any subset A of X , we can construct a unique real valued function $f_A(x)$, called characteristic function defined over the universe of discourse X such that:

$$\begin{aligned}
 f_A : X &\rightarrow [0, 1] \text{ such that} \\
 f_A(x) &= 1 \text{ if } x \in A \text{ is true} \\
 &= 0 \text{ if } x \in A \text{ if false.}
 \end{aligned} \tag{1}$$

However, in real physical world, the above assumption ($x \in A$ or $x \notin A$) may not be true. In order to exemplify let us consider the “Class of new cars”, “Class of short men”, “Class of new, high buildings”, “Class of all real numbers which are much greater than 1”, “Class of expensive bike”, “Class of highly contagious diseases”, “Class of beautiful flowers”, “Class of the boy who resembles his father”, the class of real number “approximately equal to 3”, “Class of sunny days”, We observe that due to the presence of the terms ‘new’, ‘short’, ‘high’, ‘much’, ‘expensive’, ‘highly’, ‘beautiful’, ‘sunny’, ‘approximately’, in the formation of above classes some kind of imprecision or ambiguity or vagueness arises in deciding whether an individual element in the context is an element of the class or not. Indeed classes of such types does not constitute a set in usual mathematical sense [9]. Sets of this type very often involve some adjectives, verbs and adverbs or some combination thereof which are not sharply defined in their descriptions. Numerous other examples may be found in every branch of science as well as in writings and daily conversations. In fact, most of the classes of objects encountered in the real physical world are this ‘fuzzy’ not sharply defined type. They do not have precisely defined criteria of membership. In such classes an object need not necessarily either belong to or not belong to a class, there may be intermediate grades of membership. In other words we can say the transition from member to non- member appears gradual rather than abrupt. Thus, the fuzzy set introduces vagueness with the aim of reducing complexity by eliminating the sharp boundary dividing members of the class from non- members.

A fuzzy set can be defined mathematically by assigning to each individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is “similar” or “compatible” with the concept represented by the fuzzy set [2]. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1. This concept of a fuzzy set, which is a “class” with a continuum of grades of membership or we can say that a fuzzy set on a set X is sorted of generalised “Characteristic function” on X , whose degree of membership may be more than “yes” or “no”. Thus, a fuzzy set representing our concept of sunny days might assign a degree of membership 1 to a cloud of 0 percent, 0.8 to a cloud cover of 20 percent, 0.4 to a cloud cover of 30 percent and 0 to a cloud cover of 75 percent. These grades signify the degree to which each percentage of cloud cover approximates our subjective concept of

sunny days and the set itself models the semantic flexibility inherent in such a common linguistic term. Because full membership and full non-membership in the fuzzy set can still be indicated by the values of 1 and 0, we can consider the “crisp” set to be restricted case of the more general fuzzy set for which only these two grades of membership are allowed. The notion of fuzzy sets can be represented: Let X denote a universal set also referred as a field of reference or universe of discourse. Then, the membership function f_A by which a fuzzy set A is usually defined has the form,

$$f_A : X \rightarrow [0, 1] \text{ where } [0, 1] \quad (2)$$

denotes the interval of real numbers from 0 to 1, inclusive. The value of f_A at x , $f_A(x)$ is 1 or 0 according as x belongs or does not belong to A . When A is a fuzzy set, then the nearer the value of $f_A(x)$ to 0, the more tenuous is the membership of x in A , with the “degree of belonging” increasing with increase in $f_A(x)$.

3. MEDICAL LITERATURE

3.1. Sources of Uncertainty

The complexity of medical practice makes traditional quantitative approaches of analysis inappropriate. In medicine, the lack of information and its imprecision, and many times, contradictory nature are common facts [4]. The sources of uncertainty can be classified as follows:

Information about the patient: Medical history of the patient, which is usually, supplied by the patient and/or his/her family. This is usually highly subjective and imprecise.

Physical examination: The physician usually obtains objective data, but in some cases the boundary between normal and pathological status is not sharp.

Results: Results of laboratory and other diagnostic tests are also subject to some mistakes and even to improper behaviour of the patient prior to the examination.

Symptoms: The patient may include simulated, exaggerated, understated symptoms, or may even fail to mention some of them.

Classification: We stress the paradox of the growing number of mental disorders versus the absence of a natural classification. The classification in critical (ie, borderline) cases is difficult, particularly when a categorical system of diagnosis is considered.

3.2. Fuzzy Logic and Medicine

Fuzzy logic plays an important role in medicine. Some examples showing that fuzzy logic crosses many disease groups are the following.

- Fuzzy information granulation of medical images. Blood vessel extraction from 3-D MRA Images.
- Awareness Monitoring and Decision-Making for General Anaesthesia.
- Acquisition of Fuzzy Association Rules from Medical Data.
- Fuzzy logic in a decision support system in the domain of Coronary heart disease risk assessment.
- A model-based temporal abductive diagnosis model for an intensive coronary care unit.
- A Fuzzy Model for Pattern Recognition in the Evolution of Patients
- To predict the response to treatment with citalopram in alcohol dependence [3].
- To analyze diabetic neuropathy and to detect early diabetic retinopathy.

- To calculate volumes of brain tissue from magnetic resonance imaging (MRI), and to analyze functional MRI data.
- To assist the diagnosis of central nervous systems tumors (astrocytic tumors).
- To discriminate benign skin lesions from malignant melanomas
- To improve decision-making in radiation therapy.
- To visualize nerve fibers in the human brain.
- To represent quantitative estimates of drug use.

Many other areas of application, to mention a few, are to study fuzzy epidemics, to make decisions in nursing, to overcome electro acupuncture accommodation. The diagnosis of disease involves several levels of uncertainty and imprecision, and it is inherent to medicine. A single disease may manifest itself quite differently, depending on the patient, and with different intensities. A single symptom may correspond to different diseases [5]. On the other hand, several diseases present in a patient may interact and interfere with the usual description of any of the diseases. The best and most precise description of disease entities uses linguistic terms that are also imprecise and vague. Moreover, the classical concepts of health and disease are mutually exclusive and opposite. However, some recent approaches consider both concepts as complementary processes in the same continuum. According to the definition issued by the World Health Organization (WHO), "health is a state of complete physical, mental, and social well-being and not merely the absence of disease or infirmity". The loss of health can be seen in its three forms, disease, illness, and sickness.

To deal with imprecision and uncertainty, we have at our disposal fuzzy logic. Fuzzy logic introduces partial truth values, between true and false. According to Aristotelian logic, for a given proposition or state we only have two logical values, true-false, black-white, 1-0. In real life, things are not either black or white, but most of the times are grey. Thus, in many practical situations, it is convenient to consider intermediate logical values. Let us show this with a very simple medical example. Consider the statement "you are healthy". Is it true if you have only a broken nail? Is it false if you have a terminal cancer? Everybody is healthy to some degree h and ill to some degree i [8]. If you are totally healthy, then of course $h = 1, i = 0$. Usually, everybody has some minor health problems and $h < 1$, but $h + i = 1$. In the other extreme situation, $h = 0$, and $i = 1$ so that you are not healthy at all (you are dead). In the case you have only a broken nail, we may write $h = 0.999, i = 0.001$; if you have a painful gastric ulcer, $i = 0.6, h = 0.4$, but in the case you have a terminal cancer, probably $i = 0.95, h = 0.05$.

Uncertainty is now considered essential to science and fuzzy logic is a way to model and deal with it using natural language. We can say that fuzzy logic is a qualitative computational approach. Since uncertainty is inherent in fields such as medicine and fuzzy logic takes into account such uncertainty, fuzzy set theory can be considered as a suitable formalism to deal with the imprecision intrinsic to many biomedical and bioinformatics problems. Fuzzy logic is a method to render precise what is imprecise in the world of medicine.

4. FUZZY SYSTEM IDENTIFICATION

The concept of a mathematical model is fundamental to system analysis and design which require the representation of systems phenomenon as a functional dependence between interacting input and output variables. Mathematical models are essential for prediction and control purposes [6]. Conventionally, a mathematical model for a system is constructed by analysing input-output measurements from the system. These numerical measurements are important because they represent the behaviour of the system in a quantitative fashion. Very often, there exist another important information source for many engineering systems, knowledge from human experts. This knowledge known as linguistic information, provides qualitative instructions and descriptions about the system and is especially useful when the input and output measurements are difficult to obtain.

Fuzzy models are capable of doing this kind of information naturally and conveniently, while conventional mathematical models usually fail to do so. Moreover, it is interesting to note that fuzzy models have the same ability to process numerical information as conventional models, if such information is available. Being able to deal with linguistic information and numerical information is one of the important property of fuzzy models. The second important property of fuzzy models is their ability to handle non linearity. It is well known fact that most

engineering systems are non linear to some extent. Interpretability is another salient feature of fuzzy models. A fuzzy model has a transparent model structure. Each rule in the model acts like a “local model” in the sense that it only covers a local region of the input-output space, and its contribution to the whole output of the model is easily understood. Fuzzy system identification consists of three basic sub problems, specification, estimation and validation. This is depicted in the Figure. 1.

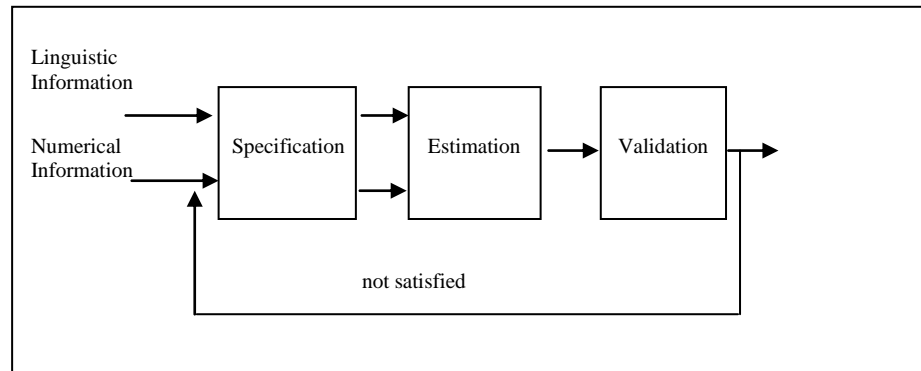


Figure. 1 A System Identification

Specification involves finding the important input variables from all possible input variables, specifying membership functions, partitioning the input space, and determining the number of fuzzy rules comprising the underlying model. Parameter estimation involves the determination of unknown parameters in the model using some optimization method based on both linguistic information obtained from human experts and numerical data obtained from the actual system to be modelled. Specification and estimation are interwoven, and neither of them can be independently identified without resort to other. Validation involves testing the model based on some performance criterion. If the model cannot pass the test, we must modify the model, the model structure and re-estimate the model parameter. It may be necessary to repeat this process many times before a satisfactory model is found. The specification of a fuzzy model involves selection of input variables. The different techniques for selecting input variables are forward selection, backward selection, and best subset procedure. The choice of membership functions affect how well a fuzzy model behaves.

Empirical evaluation of different membership functions can be useful in guiding the choice of membership functions. Fuzzy models are constructed using triangular, bell shaped and Gaussian membership function. Triangular membership function is inferior to bell shaped and Gaussian membership function. These membership functions are compared with other membership functions especially, with the sine function $\sin(x)/x$, based on how closely the resultant fuzzy models approximate the real systems. More extensive empirical investigation is needed in this area. Parameter estimation problems, in general, involve the optimization of antecedent membership parameters and the consequent parameters. In evaluating fuzzy models cross validation, residual analysis, and information-theoretic criteria are employed.

4. SUMMARY OF FINDINGS

4.1. Estimation of the risk of Smoking

We know that fuzzy control work on fuzzy inference. The idea of fuzzy inference is applied to fuzzy expert system. It is now being used not only in engineering but in several fields of social sciences. In this light we will show the example based on consistencies of fuzzy sets. Let us consider the following situation of smoking which is one of the vital cause behind heart attack, cancer etc. among human beings.

A doctor thinks that smoking less than 10 cigarettes a day is not harmful for health (safe).

He presumes that smoking more than 50 smokes a day is definitely (absolutely) harmful (dangerous).

The doctor believes that smoking 20 to 30 smokes per day is potentially harmful (suspicious). The doctor understanding about smoking can be illustrated with the help of fuzzy sets as shown in the Figure. 2.

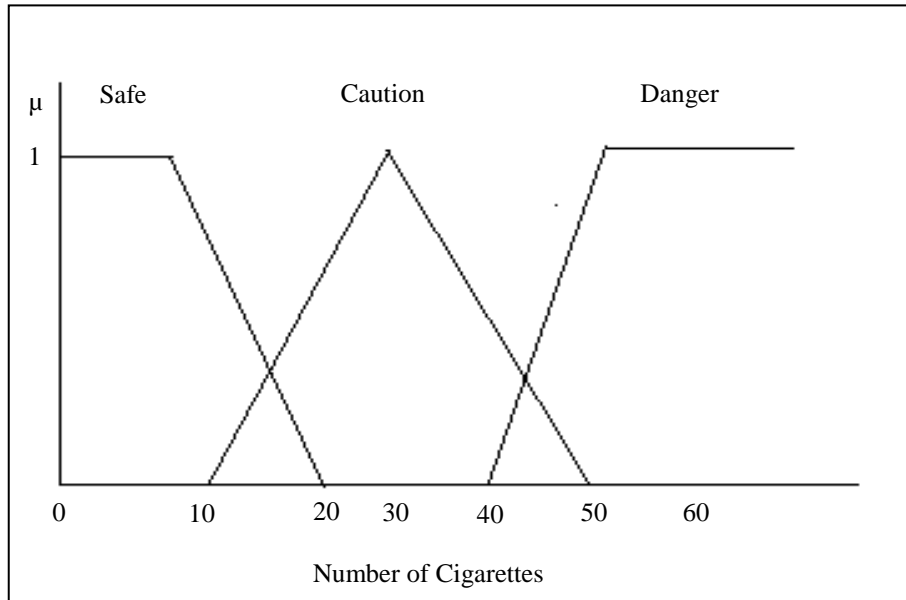


Figure. 2 Membership function of risk of smoking

On the horizontal axis i.e. on the x-axis, we express the number of smokes a day, and the membership function that expresses the risk of smoking has been indicated on the vertical axis i.e. on y axis between the real number 0 and 1. Let us suppose that Ajay’s satisfaction of smoking is given by the fuzzy set A. That is he is satisfied if he has about 10 to 20 smokes a day. Also we have Binod’s fuzzy set B, which means Binod’s requires about 40 to 50 smokes a day. The doctor’s diagnosis (understanding) about smoking is given by the points of agreement of each of the fuzzy sets, safe, caution and danger. We know that the points of agreements are given by the highest value at the intersection of two fuzzy sets. The intersection of two fuzzy sets A and B is defined as,

$$(A \cap B)(X) = \min \{ A(X), B(X) \} \tag{3}$$

for all $x \in X$, where X is the universal set, A(X) and B(X) are the membership functions. We may infer that Ajay has the safety degree of 0.9, the degree being Caution of 0.9, and the danger degree of 0.0. Also, we have Arun fuzzy set we infer that Arun’s danger degree is 0.85 and his safety degree is almost zero. Here we note that the membership function is used to give the diagnosis that can be provided by the statistics or by the doctor’s subjectivity. This example shows a way to give a subjective diagnosis, which might be too simple for real use. If the patient is required to be classified, we can take the label whose value is the highest. Here we also observe that in this example, the horizontal axis gives continuous numbers.

4.2. Estimation of the risk of Alcohol drinking

Alcohol drinking and Cigarette smoking during adolescence have been shown to be associated with a greater possibility of concurrent and future substance-related disorders. In order to report patterns of drug use and to describe factors associated with substance use in adolescents, a cross-sectional survey was carried out in a representative population sample of 3000 adolescents, aged 12 to 17 years, from Bangalore, silicon city of India to Bombay, a metro and commercial capital of India. The original survey covered the use of alcohol, tobacco, illicit drugs, and other psychoactive substances. For tobacco smoking and alcohol drinking, each subject of the population sample can be assigned a fuzzy degree of addiction (or risk use). With respect to the other fuzzy variable, if you drink no alcohol, the degree of this variable is 0. If you drink more than 75 cc of alcohol per day, the degree of alcoholism is 1. For 25 cc/d, the degree could be 0.4 and for 50 cc/d, 0.8. Suppose you correspond to the fuzzy set $\lambda = (1, 1)$, have recently had some health problems, and your physician has advised you to reduce your consumption of cigarettes and alcohol by half. The ideal situation for your health is, of course, the point $\mu = (0, 0)$, but it is possibly difficult to achieve. Since uncertainty is inherent in fields such as medicine, fuzzy logic takes into account such uncertainty to render precise, overcome imprecise in the world of medicine.

5. CONCLUSIONS

In this interesting paper the author presents a summary of the basic concepts and techniques underlying the application of fuzzy set theory to solve practical problems related to health. The study finds a number of examples relating to its use as a computational system for dealing with uncertainty and imprecision in the context of evaluation of risk associated with smoking and drinking habits. Health is a vital indicator of human development that will enable every individual to lead a social and economically productive life. Through the process of fuzzification, we gain greater generality and enhanced ability to model real world problems and higher expressive power to some extent for respective areas of non fuzzy mathematics.

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