CRITICAL CHAIN PROJECT SCHEDULING: UTILIZING UNCERTAINTY FOR BUFFER SIZING

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ABSTRACT

In this paper, we introduce a method for buffer sizing in Critical Chain project scheduling. Proposed approach considers lognormal distribution for modeling activities' execution time. The inherent uncertainty of activities considered as a main factor to size the buffer in proposed method. For comparing with traditional methods we used problems presented in literature. The results depict that proposed method outperforms critical chain buffer sizing method.

Keywords: Project management, Buffer sizing, Critical chain, Risk management

1. INTRODUCTION

After presenting the Theory of Constraints by Goldratt [1], it was first applied to production systems. Later on, concerning the philosophy of this theory and its continuous improvement strategy, mentioned in five stages, it is propagated to other scientific branches such as project management.

Algorithms and methods derived from the Theory of Constraints caused change in production industries and project management but lack of mathematical relations and statistical reasoning in the steps of the suggested process is the present problem.

In Critical Chain method, which in fact derived from propagation of Theory of Constraints in project management, all the safety times, correlated with critical activities, are moved to the end of project and considered as a project buffer. In addition to project buffer, Feeding buffers are cited in the project to protect the Critical Chain, in any time that non-critical activities are connected to critical chain. Resource buffers are characterized as warning systems or reminders that make sure the resources are ready when it is time to work on a critical task

Subsequent to the publication of *Critical Chain*[2] ,recent books Newbold,[3] ; Leach,[4] ,articles (for example , Cabanis and Brewin,[5]; Glober and son,[6]; Maylor,[7]; Patrick,[8]; Pinto,[9]; Rand,[10,11]; Steyn,[12,13]) , Improvement papers (for example, Wei et al., [14]; Tukel et al.,[15]), Investigation paper (for example, Vonder et al.[16] Herroelen and Leus,[17]) have been written in this context.

In addition Herroelen and Leus [18] provide an extensive review of the scheduling literature, and a discussion of reactive and robust project scheduling. Elmaghraby et al. [19] introduce an application of TOC in resource constrained project scheduling.

The first presented method to size the buffers succeeding Goldratt's theory is a Cut & Paste method(Standard CCPM) that is based on using 50% of the safe estimates as task durations (Goldratt [2]). After determination of critical chain the half of sum of the critical (feeding) tasks will be used as project (feeding) buffer. This method attempts to size the buffers, using compound risk of activities.

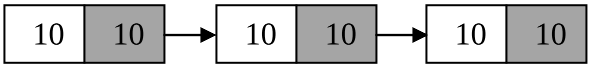
In 1998, Newbold [3] presented a method to size the buffer in Critical Chain in which, for the first time, the time of doing an activity is described through the lognormal distribution function. He claims that buffers are a kind of aggregation of the risk encountered along the chain of activities. However, Herroelen et al [20] disagree with Newbold's claim regarding the simplification of critical chain implementation.

As mentioned before, lack of dynamic mathematical relations to size the buffers causes the allocation of a constant or using a fixed coefficient in all projects. Considering the present competitive conditions in project environments, this causes to miss the commercial opportunities.

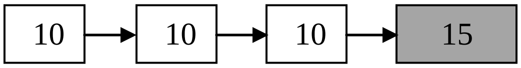
The remainder of paper is organized as follows; a statement of problem and motivation of proposed approach are given in next section. Section 3 describes utilizing uncertainty followed by introduction to use the risk indicators for buffer sizing. Application of lognormal distribution for completion time of activities and a novel procedure for parameter determining is introduced in Section 4. Section 5 is devoted to proposed algorithm. To test the performance of the proposed algorithm, numerical experiments are given in section 6. The paper concluded in section 7.

2. PROBLEM STATEMENT

Considering activities' time consist of safety time besides real time estimation, critical chain scheduling approach utilizes just real time and suggests aggregating all safety times into project and feeding buffers. To size buffers, after determining critical chain, feeding activities' safety time is summed and half of this sum will be used as feeding buffer. Similar to feeding buffer, half of all critical activities' safety time constitute project buffer. For illustration, consider the following chain contain three activities:



As depicted each task's safe estimate is given by 20 days. To size project buffer, safety time is cut which yields 10 days as an average execution time, afterwards half of all safety time is summed up (5+5+5=15) to use as project buffer.



Its ease of use makes it a common approach for buffer sizing. Despite its simplicity, the size of buffer increases linearly with the length of activity chain. For instance, a two year buffer will be considered for a four year project. Especially, for projects in a low risk environment this is a misleading protection. While in high risk situations it may be not sufficient.

Accordingly, Cut & Paste method suffers lack of dynamic relations which can size the buffer based on inherent risk of activities.

3. UTILIZING UNCERTAINTY

The problem tackled in this paper is the buffer sizing in critical chain scheduling; we provide a sketch of the relevant uncertainty indicators which can be used for buffer sizing in Sect. 3.2. Beforehand, Uncertainty in statistical distributions is introduced in detail in Sect. 3.1. An analysis of defined procedures is given in Sect. 3.3.

3-1. Uncertainty in statistical distributions

In distribution function with symmetrical shape, like normal distribution, mean and median quantities are conformed and have equal quantities, now if distribution has left or right skewness, the median value would be more or less than mean in order. Therefore, in lognormal distribution with right skewness, the value of mean would be more than median's.

The difference which exists between mean and median is in direct interaction with σ quantity so that with increasing σ, this difference would also increase. Or in other words with increase in risk, σ would increase and subsequently the difference between mean and median would increase.

If D shows this difference, the quantity of D would be represented as follows:

$$D = E(X) - Median(X) \quad (9)$$

Clearly the dimension of D is time and this difference can be mentioned as a time indicator of the risk for activity X that have a lognormal distribution.

3-2. Uncertainty indicators

The combination of risk measurement indicators such as Coefficient of Variation, Skewness and Kurtosis used with D can be used to introduce fit relations to size the buffers. The definition of risk measurement indicators is as follows.

Coefficient of Variation. In statistical methods, CV is calculated by the proportion of the standard deviation to the mean and as the above proportion increases for a probability function, the function has more dispersion in comparison with its mean.

CV can be calculated as follows:

$$Coefficient \quad of \quad Variation = \frac{\sqrt{Var(X)}}{E(X)} \qquad (10)$$

Skewness. This indicator is used to measure the lack of symmetry of a distribution, so when the measure of skewness increases, the distant numbers have more distance from the mean. The numerical amount of skewness is calculated as follows:

$$Skewness = \frac{E\left[\left(X - \mu\right)^3\right]}{\sigma^3} \qquad (11)$$

Kurtosis. Kurtosis is a measure of the flatness or peaked nature of a distribution relative to a normal distribution. This indicator is used to measure the frequency or the times of occurring much bigger or smaller numbers than the mean. As a result, the more is the kurtosis of a distribution function, the more would be the risk. In statistical issues the kurtosis measure of distribution function is calculated through the following formula:

$$Kurtosis = \frac{E\left[\left(X - \mu\right)^4\right]}{\sigma^4} \qquad (12)$$

The following relations are defined through the above indicators for buffer sizing:

$$1)\quad \frac{\sqrt{Var\left(X\right)}}{E\left(X\right)} \times D \quad (13)$$

$$2)\quad Skewness \times D \quad (14)$$

$$3)\quad Kurtosis \times D \quad (15)$$

3-3. Analyzing procedures

The three above relations give us different quantities for the size of buffer. To find out the weakness and strength of each of introduced formulas, they used to size the amount of buffer for 100 activities with different parameters, the following results are obtained through simulation:

First of all, for different quantities of μ, σ and using the (13) the size of buffer is determined. Through the simulation, the activities' execution times are more than the buffer in about 35 to 40 times. Or in other words, the amount of protection of each activity by this buffer is 60% to 65%.

Using (14), (15), for circumstances where σ is constant, the measure of protection for various activities without different quantity of μ, would be a fixed percentage but if σ varies, this quantity of protection would change that in this case the level of protection increases with the increase of σ. Of course the level of protection in (15) is more than (14) in all conditions.

Considering the results of applying each of the formulas, it is shown that single usage of the formulas, would cause disorder in the structure of sizing the buffers. Consequently it is logical to combine the formulas to balance the disorder and obtain an acceptable amount of protection at least 90% which will be discussed in Section 5.

4. APPLYING LOGNORMAL DISTRIBUTION

In this paper we suppose that activities' duration is a random variable which follows lognormal distribution. Therefore, lognormal distribution is studied in Sec. 4-1. Afterwards, supposing one uses lognormal distribution in simulation of the activities completion time, a novel procedure is described in Sec. 4-2 and 4-3 to determine lognormal parameters $\left(\mu,\sigma\right)$.

4-1. Lognormal distribution

Lognormal distribution function is derived from Normal distribution. So that if $Y \sim N\left(\mu,\sigma\right)$ thus $X = e^Y$ contains lognormal distribution with μ and σ parameters which are named scale parameter and shape parameter, respectively. But these quantities do not show the mean and variance of distribution, the mean and variance are themselves functions of μ and σ. In [4], [5] lognormal distribution introduces as a proper function to describe the activity completion time.

If $X \sim Ln\left(\mu,\sigma\right)$ so X characteristics would be as follows [21]:

$$E(X) = e^{\mu + \frac{1}{2}\sigma^2} \qquad (1)\ (2)$$

$$Var(X) = e^{2\mu + \sigma^2}\left(e^{\sigma^2} - 1\right) \qquad (3)$$

$$(4)$$

$$Median(X) = e^{\mu}$$

$$Mode(X) = e^{\mu - \sigma^2}$$

4-2. Determination of scale parameter

In project management, through Critical Chain method, the probability of ending each activity by the estimated time is 50%, whereas in statistical issues the quantities of X , in lieu of which the following formula is correct, are considered as the median.

$$P(X \geq x) = P(X < x) = 0.5$$

In results, supposing to have the median time of each activity, the quantity of μ can be calculated through the following formula:

$$(5)$$

$$Median(X) = e^{\mu}$$

$$(6)$$

$$\mu = Ln(Median(X)) \qquad (7)$$

The first part is the formula of calculating the median in lognormal distribution according to which, the quantity of scale parameter is determined for activity X . This formula can be used to calculate μ quantities for all activities supposing to have their median time.

4-3. Determination of shape parameter

According to subjects mentioned, a span of σ can be used for which, distribution function is in proportion with the characteristics of the considered variable (activity completion time).

In lognormal distribution, σ is the parameter of the shape. The reason of such naming is the direct effect of the above parameter upon the figure of distribution.

The considered variable is the time of performing an activity and as demonstrated in Newbold [3] the proportional shape for this variable would be the non-symmetrical bell shape with right skewness. Consequently, by considering that the shape of lognormal distribution changes to the shape of exponential distribution in the case of $\sigma > 1$, the primary span for shape parameter is set to $0 < \sigma < 1$.

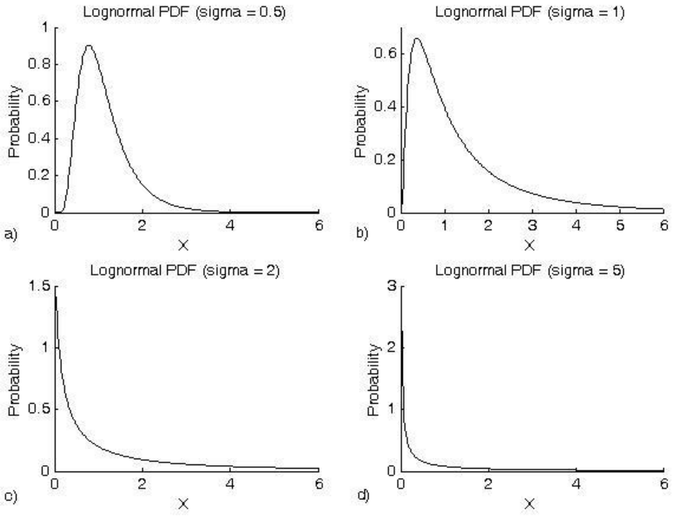


Figure 1. Lognormal Distribution (shape vs. shape parameter)

Generally in project environments, in specific, the probability of ending an activity during a time twice as much as the time of median, should be an acceptable amount. In this paper without loss of generality we set this probability to 0.9. It is in the case that in the defined space of σ, though the shape of the distribution would be as desired, but if $\sigma > 0.5$, the probability of ending the activity with parameters $\mu = Ln\left[Median\left(X\right)\right]$ and $\sigma > 0.5$ in the time span of $0 < T \le 2Median$ would not be an acceptable quantity.

Table.1 shows the probable quantities of $P\left(X < 2Median\right)$, supposing various quantities for σ.

σ	$P\left(X < 2Median\right)$	$P\left(X > 2Median\right)$
0.6	0.88	0.12
0.7	0.84	0.16
0.8	0.81	0.19
0.9	0.78	0.22
1	0.76	0.24

Table1. Value of $P\left(X \le 2.Median\right)$

So, to make more proper domain for shape parameter, a new range for σ is as follows:

$$0 < \sigma \le 0.5 \quad (8)$$

In the new range, if $\sigma = 0$ the distributions function would be changed to a constant value. This case can be considered to be used for the activities which have standard fixed time.

As shown in Table.2 the secondary range is converted to a discrete range between 0.1 and 0.5. Now with due attention to the risk of each activity, a corresponding quantity for σ can be obtained.

Risk	σ
Very low	0.1
Low	0.2
Medium	0.3
High	0.4
Very high	0.5

Table 2. Value of σ

5. ADAPTIVE RISK-BASED PROCEDURE (ARBP)

The stages of proposed algorithm to buffer sizing would be as follows:

1. At first we calculate the buffer amount of each activity using the three presented formulas ((13), (14) and (15)).
2. Arrange the obtained numbers in a descending order so that:

$$a\left(1\right) > a\left(2\right) > a\left(3\right)$$

3. In this part, calculate the numerical amount of the following convex combination:

$$\frac{a\left(1\right)-a\left(2\right)}{a\left(1\right)-a\left(3\right)} \times a\left(1\right) + \frac{a\left(2\right)-a\left(3\right)}{a\left(1\right)-a\left(3\right)} \times a\left(2\right) \quad (16)$$

The obtained formula can be applied as a dynamic formula to size the buffer. The present problem is to size the project buffer and feeding buffer. Therefore the forth step would be defined as follows:

4. After calculating the buffer of each activity, the size of project buffer is obtained by adding up the buffer of critical activities and the size of feeding buffer is determined through adding up the buffer of activities of each of the un-critical chains.

5-1. Upper and lower bound

The above algorithm has a more proper structure in comparison with other formulas. By using the simulation to evaluate the mentioned algorithm, the desired condition would not exist in two cases. Here, these cases and related solution are presented:

1. The case in which the total critical activities are in low level and consequently the weighted average of σ is also close to 0.1. Mathematically, the weighted average of σ for n activities defined as follows:

$$\bar{\sigma} = \frac{\sum_{i=1}^{n}\left[\sigma_i \times Median\left(X_i\right)\right]}{\sum_{i=1}^{n}\left[Median\left(X_i\right)\right]} \quad (17)$$

In this case the resulted buffer does not provide the desired protection of the project that is 90%.

2. The case in which the risk of the total activities is in a high level and so the average measure of σ is close to 0.5.

In this case, buffer absolutely protects the project (100% protection); however a certain protection could be obtained by inserting an ample buffer in any project. Thus, percentage of protection without any concern of buffer's length may deceive project manager and team.

The reason why these cases occur is in using the third formula(15). Because the buffer determined by this indicator for the quantities of $\sigma \to 0.5$ is more than needed and also for $\sigma \to 0.1$ the obtained buffer is close to the quantities of the (13) and (14) and as a result a decreasing buffer is determined.

The size of the buffer does not seem logical in neither the first case nor the second one because even if we suppose the risk of all activities in a project is in a low level, the probability that all the activities are done by the appointed time, would be a small quantity, and in the case that the risk of all activities is in a high level, the probability that all activities are not complete by the appointed time, would also be a small quantity. Therefore, to avoid this problem, it seems necessary to appoint a domain for the third factor(15).

To do so, upper bound and lower bound are determined as follows: By considering the 90% protection, the lower bound is determined through the following formula:

$$LB = F^{-1}\left(0.9, \tilde{\mu}, \bar{\sigma}\right) \quad (18)$$

Where F^{-1} is the inverse of lognormal cumulative distribution function and with supposing n activities $\tilde{\mu}$ equals to:

$$\tilde{\mu} = Ln\left(\sum_{i=1}^{n} Median\left(X_i\right)\right)$$

But it should be considered that the quantity of lower bond is big when the total risk of activities is high. So considering the point that a project is normally a combination of activities with both high and low risks, it would have a risk of average level. At last, consider σ as equal to 0.3 and the measure of protection as 90% and calculate a controlling factor for the lower bound:

$$UB = F^{-1}\left(0.90, \tilde{\mu}, 0.3\right)$$

In this condition, it is expected that projects reach to 90% protection by balancing the obtained quantity using the convex combination. Figure.2 shows the procedure.

(19)

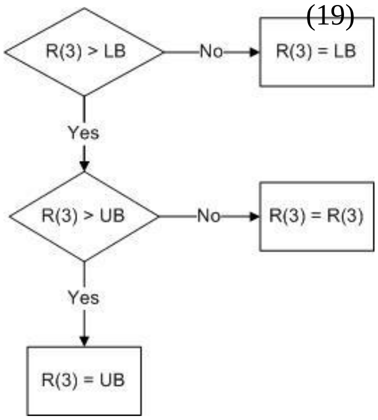


Figure 2. Lower bound and upper bound for R(3)

Where, R(3) means the obtained quantity by (15).

5. NUMERICAL EXPERIMENTS

In this section, through the obtained relations, some problems that are mentioned in literature are analyzed by using proposed algorithm and compared to standard CCPM method. Figures.2-7 illustrates the network of problems. Each square in network shows name, median time and needed resource of activity. For example in Figure.3 the first square means that the median time of activity E1 is 15 and the needed resource is E.

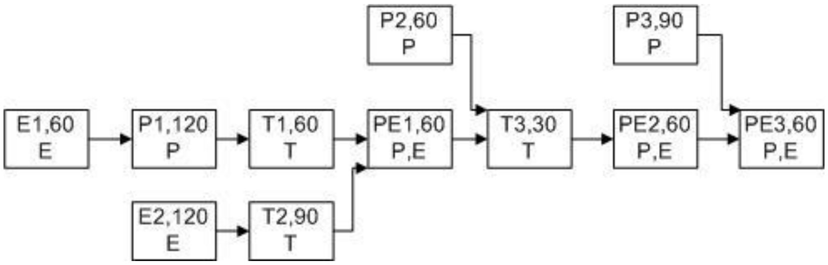


Figure 3. Problem.1

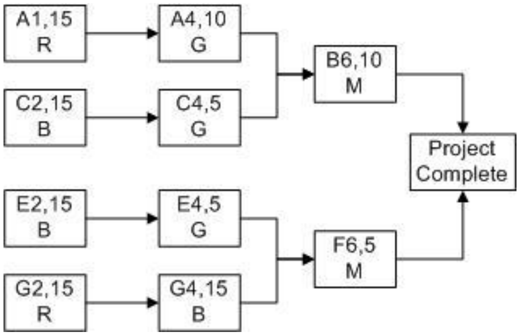


Figure 4. Problem.1

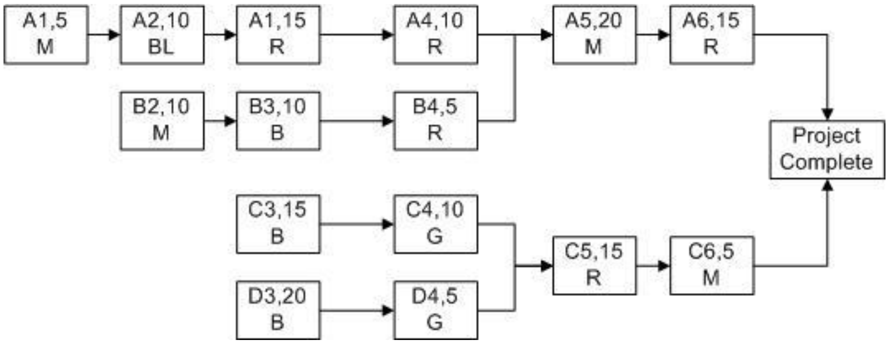


Figure 5. Problem.2

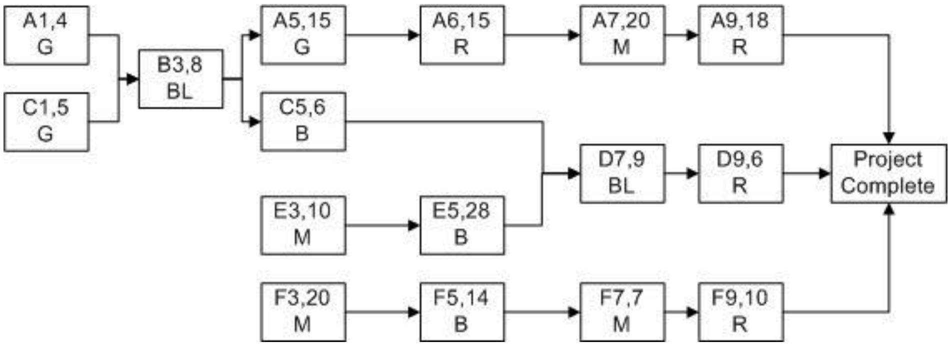


Figure 6. Problem.3

All the presented problems in this part are first solved using the CCPM+ software (add-ins for MSP), which is one of the available software to study the project through critical chain method, and then each of the problems is simulated with Monte-Carlo simulations.

In each of these problems, through Monte-Carlo simulation, the random quantities generate for σ by using Excel software. Therefore, Table (3) is given to the software as an input to produce the random quantities.

σ	$f(\sigma)$
0.1	0.2
0.2	0.2
0.3	0.2
0.4	0.2
0.5	0.2

Table 3. Value of σ for simulation

By 100 times of simulation for σ, each of the problems turns in to 100 problems with similar activities and different σ for each activity and then for each minor problem virtually executed 100 times.

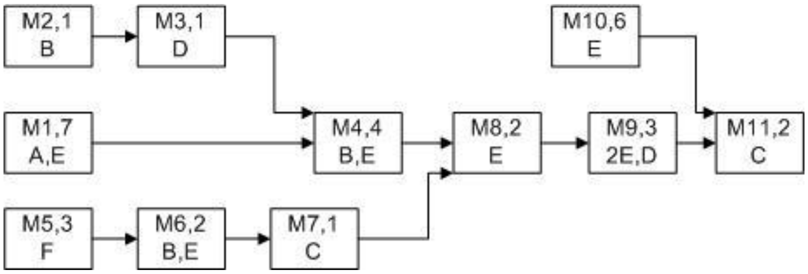


Figure 7. Problem.5

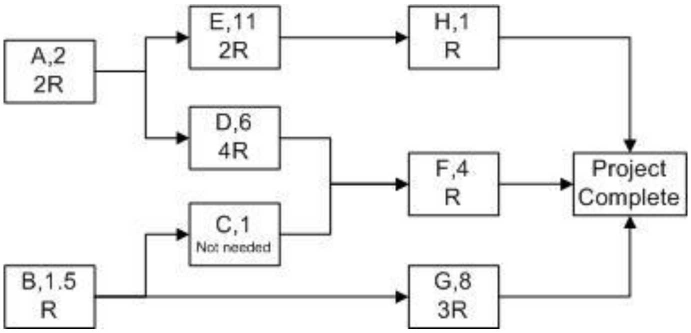


Figure 8. Problem.6

For this reason, to obtain completion time of each activity in each execution the following formula is used to produce random quantities of lognormal distribution:

$$D(X) = LN(\mu_x, \sigma_x) \sim e^{\mu_x} \times e^{[\sigma_x \times N(0,1)]}$$

Where $D(X)$ is the completion time for activity X .

In many cases, by decreasing the total risk of activities, the size of the suggested buffer decreases to ½ of the buffer in the critical chain method, whereas the measure of protection would not change more than 3 to 5 percents. Table.4 focuses on the size of project buffer. So for each problem the minimum and maximum of project buffer obtained by new algorithm compared to standard CCPM are shown.

		Problem #					
		1	2	3	4	5	6
Proposed Algorithm	Min	141	11	24	24	4	3
	Max	310	29	49	55	11	9
CCPM		270	25	42.5	47.5	9	7.75

Table 4. Size of project buffer through the methods

	Problem #					
	1	2	3	4	5	6
Proposed Algorithm	94%	90%	92%	96%	90%	91%
CCPM	95%	81%	95%	95%	87%	87%

Table 5. Minimum protection in CCPM and proposed method

The project buffer in each of six problems by the CCPM method is constant and equal to the half of critical chain duration. Proposed method suggests the buffers according to the risk of activities that causes project buffer varied from minimum to maximum. The minimum protection of each method through the simulation is given in Table.5. As shown the CCPM method protection is in an interval between 81% and 95% but the proposed method holds the minimum protection at 90%.

It appears from the results that the presented algorithm, dynamically, sizes the buffer considering the risk of activities. Of course in some cases of increasing the size of buffer, the measure of protection would not change, in these conditions, the unprotected cases are in a small distance from the suggested buffer. In these cases, practically, we can protect the project with doing slight changes, if needed, whereas in critical chain method, we have to make basic changes in the time of project delivery to size the buffer if the unprotected cases occur. Table.6 shows the coefficient of variation of lack of protection in the problems through the methods.

	Problem #					
	1	2	3	4	5	6
Proposed Algorithm	0.74	0.23	1.04	1.04	0.67	0.54
CCPM	1.14	0.59	1.15	1.21	0.83	1.01

Table 6. Comparison of C.V of protection in two methods

Obviously, in the all solved problems, the coefficient variation of lack of project protection percentage, in the presented method, is by far less than CCPM method. Or in the other words the presented method has less risk than CCPM method in measure of lack of project protection.

6. CONCLUSION

The standard CCPM uses the half of critical chain to size the project buffer. Implementation of this procedure forces the buffer which is more than needed to project. In this paper we use lognormal distribution function for task completion time and proposed a new algorithm to size the buffers. It appears from the results that compared to standard CCPM, proposed algorithm size the buffers dynamically and in comparison of CV of protection it overcomes the standard CCPM.

An extension of this study would be to include other statistics distribution for task completion time. For activities in projects the distributions should satisfy the main characteristics of completion time that are mentioned in paper.

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