PHYSICAL DOCTRINE OF TURBULENCE – A REVIEW

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ABSTRACT
This paper reviews the physical background, the formalism and the outcomes of the approach to the treatment of turbulence problems developed by the author (called the physical doctrine of turbulence). The approach is based on the classical conception of turbulence by L. F. Richardson and A. N. Kolmogorov as of a form of organization of motion of fluids characterized by the incessant restoration of their hierarchic eddy structure through the cascading process maintained by the divergence of scales of gain and disposal (dissipation) of the motion energy. The explained formalism is built up on two principles and on one method. The review is concentrated on an outcome of the approach advancing the physical background of the turbulence mechanics and enlarging its application capacity. Two particular formulations of the turbulence mechanics realizing the advantage are commented.

Keywords: turbulence, micropolar fluids, statistical physics.

1. INTRODUCTION
All turbulence treatments can be classified into three basic categories of doctrines. O. Reynolds [1] initiated the first doctrine. This doctrine identifies the turbulence with a chaotic (unordered) form of motion of fluids. The second doctrine treats the turbulence as the problem of integration of equations of classical fluid mechanics (CFM) for the large Reynolds number values set to the computational fluid dynamics (CFD). The third doctrine (henceforth, the physical doctrine of turbulence or the PDT) treats the turbulence according to L. F. Richardson [2] as a form of organization of fluid motion characterized by the incessant restoration of its hierarchic eddy structure through the cascading process maintained due to the divergence of the scales of gain and disposal (dissipation) of the motion energy in conjunction with the complementation by A. N. Kolmogorov [3] stating the motion of large-scale eddies (gaining their energy immediately from the average flow) orientated by the average flow. Unlike the first and the second doctrine, specifying the turbulence problem as the problem of closure of the average equations of the CFM and as the problem of integration of equations of the CFM, respectively, the PDT specifies the turbulence as a problem of statistical physics.

Section 2 explains the formalism of realization of the PDT. The formalism is explained basing on two principles and one method specified here as the systemic principle (SP), the kinematical-geometrical principle (KGP) and the method of structural decomposition (MSD). The SP [4-6] stems from the systemic essence of any statistical description. It is applied to clarify the relations between the classical fluid mechanics (CFM), statistical fluid mechanics (SFM) and turbulence mechanics (TM) and to explain the context of the turbulence problem within the framework of physics as a whole. The KGP was inspired by the Rieman geometry [7] compiling the properties of space in infinitesimal surrounding of a space point (explained within the Euclid geometry) and the space curvature as a global space property. It particularizes the mandated by the SP coupling (interconnection) of formulations of the SFM and the TM in agreement with the conception of L. F. Richardson and A. N. Kolmogorov (henceforth referred to as the RK conception). A substantial outcome of the KGP is the establishment of relation of the average turbulent continua to the class of micropolar fluids (MF) [8-14] with their micropolar properties reflecting local properties of the non-local eddy structure of turbulence. The MSD [4, 15-19] formulates a procedure for representing the average turbulent flow field in a scale-decomposed form invariant in respect to particular realizations of the turbulence scale-decomposition. Section 3 comments two outcomes of the PDT in formulation of the TM [4, 16, 20] specified as the theory of rotationally anisotropic turbulence (RAT theory) and as the phenomenological mechanics of turbulent flows (PMFT). Section 4 summaries the main declarations of the PDT.

2. THE FORMALISM OF THE PDT
2.1. The SP and the systemic context of the turbulence problem
2.1.1. The SP
The systemic principle (SP) [4-6] compiles the formal aspects of the statistical physics common to all statistical descriptions. The commonness rises from the systemic essence of any statistical description. By the essence the statistical description comprises (on the elementary level) three particular descriptions – the description in terms fixing the particular events, the description in terms of the probability distribution of particular events and the description in terms fixing the conditions of the probability distribution properties formation. Each of these
particular descriptions is provided with the competence not carried by other two and forms together with the other two a system of three particular descriptions. As a member of the system each particular description obtains in the system certain systemic properties not revealed if it is considered independent from the system. As an example, the affiliation to a system ascribes a particular event with the quality of randomness and couples (interconnects) the formulation of properties of a probability distribution and of the conditions of formation of properties of the probability distribution.

The explained elementary level of the systemic description can be expanded to the description of situations with a hierarchy of conditions of formation of probability. A substantial feature of the expanded description is that particular descriptions with similar locations in the system organization also carry certain similarities in their formulation.

The PDT utilizes the SP to specify the turbulence problem in the mechanical and physical contexts.

2.1.2. Elementary level of systemic description of fluids motion

![Diagram](image)

*Figure 1. Elementary level of the systemic description of motion of fluids compiling the classical fluid mechanics (CFM), the statistical fluid mechanics (SFM) and the turbulence mechanics (TM) as the elements of three theories with divided competence.*

Figure 1 particularizes an application of the SP to the setup of systemic description of fluids motion [6]. The description compiles the CFM, the statistical fluid mechanics (SFM) and the turbulence mechanics (TM). Delegating the competence of the turbulence description to a determination level higher than represented by the CFM it removes from the CFM the competence expected from the CFD (though, retains this competence in small surroundings of flow field points), and delegating the competence of the turbulence description to the TM and the SFM on different structural levels, declares an interconnection between the formulations of TM and the SFM. The interconnection explains the terms fixing the motion states within the TM simultaneously as the moments of the probability distribution, fixing the motion state within the SFM, and as the terms specifying the background conditions of formation of specific properties of the probability distribution. Notice, that besides compiling the CFM, the TM and the SFM, the systemic description in Figure 1 compiles also the theories connecting each pair of them (called in [4-6] the recoding theories).

2.1.3. Conjoint systemic description of fluids motion and of physical fields

Figure 2 represents an expansion of the systemic description in Figure 1 to account for also the canonical, electromagnetic and strong interaction levels [4, 6]. The systemic description in Figure 2 sets the turbulence problem into the context of physics as a whole and associates the similarities in mathematical formulations of the ED and CFM [21, 22] as well as of the descriptions of fluid turbulence and of fields of strong interaction [23] with their similar locations relative to the Hamiltonian mechanics (as one of the particular theories of the systemic description).
The PDT utilizes the systemic description in Figure 2 in two purposes. Firstly, it makes obvious the affiliation of the turbulence problem to the physical context as a whole. Secondly, it appreciates the RK conception emphasizing the features of organization of turbulent form of motion as common to the fields of turbulent fluids and strong interaction. More definitely, emphasizing the incessant restoration of eddy structure of turbulence this conception actually qualifies transmutations as a physical form of motion. In addition, it relates the turbulent flow fields to the fields with spin (reflecting the effect of eddy rotation) and ascribes the structure of the turbulent flow field with the property of scaling (realized in the turbulent media in the form of a hierarchy of scales of eddies participating in the motion). All these aspects are declared in [24, 25] also essential for understanding the properties of fields of strong interaction. Besides, as far as the similarities between the mathematical formulation of the ED and CFM increase if instead of the equations of the CFM the respective relativistic equations are considered [26], it can be expected that the relativistic theory of turbulence (if formulated) also increases the similarities between the properties of turbulent flow fields and the fields of strong interaction.

2.2. The KGP and the micro-polar properties of the average turbulent continua

The kinematical-geometrical principle (KGP) consists in characterizing the turbulent flow field in the infinitesimal surroundings of flow field points by the dyad \((v, k)\), where \(v\) is the flow velocity at a flow field point and \(k\) is the vector of curvature of the velocity fluctuation streamline passing this flow field point. The following explains two most important upshots of applications of the KGP. The first upshot consists in relating the average turbulent continua to the class of micropolar fluids (MF) [8-14] with their micro-polar properties induced by the flow field non-local eddy structure. The second upshot consists in revelation and suggestion a solution of a classical contradiction between the RK conception and the conventional setup of the TM (CTM). In the following these upshots are particularized in the listed order.

Let

\[
v' = v - \langle v \rangle,
\]

(1)

where the angular brackets denote statistical averaging, denote the fluctuating constituent of the flow velocity \(v\), \(k = \partial e / \partial s\) (in which \(e = v' / |v'|\) and \(s\) is the length of the curve of \(v'\) streamline passing the flow field point) is the vector of curvature of \(v'\) streamline passing a flow field point and \(R = k^{-2}k\) \((k = |k|)\) is the curvature radius vector corresponding to \(k\). The KGP enables to complement the average turbulent flow field characterization by the following kinematical-dynamical pair of the Eulerian flow-field characteristics.

Figure 2. Structure of a conjoint systemic description of motion of fluids and of physical fields.

Besides abbreviations utilized in Fig. 1 the following abbreviations are used: HD – Hamiltonian mechanics; SM – statistical mechanics; ED – electrodynamics; QT – statistical theory of quantum fields without spin; PSI – not formulated yet a phenomenological theory of strong interaction; SSI – statistical theory of strong interaction.
\[ \boldsymbol{\Omega} = \{ \mathbf{v} \times \mathbf{k} \} = \{ \mathbf{e} \times \mathbf{e} \} \]  
(2)

and

\[ \mathbf{M} = \{ \mathbf{v} \times \mathbf{R} \} = \{ \mathbf{R}^2 \mathbf{e} \times \mathbf{e} \}. \]  
(3)

In (2) and (3) the overdot denotes the full time derivative \( \partial / \partial t + \mathbf{v} \partial / \partial \mathbf{s} \) (in which \( \mathbf{v}' = \mathbf{v} \)) and \( \mathbf{R} = |\mathbf{R}| \). According to (2) and (3) \( \boldsymbol{\Omega} \) and \( \mathbf{M} \) characterize the average motion state of Lagrangian particles passing a flow field point. While \( \boldsymbol{\Omega} \) has the sense of average angular velocity of rotation of medium particles at a point in respect to the random curvature centres of the velocity fluctuation streamlines passing this point, then \( \mathbf{M} \) has the sense of average density (per unit mass) of the moment of fluctuating constituent of momentum with the random radius-vector \( \mathbf{R} \) standing for the arm of the moment. According to the physical (mechanical) sense of \( \boldsymbol{\Omega} \) and \( \mathbf{M} \), it is natural to connect them by

\[ \mathbf{M} = \ell^2 \boldsymbol{\Omega}, \]

(4)

which defines the average motion scale \( \ell \) respective to \( \mathbf{R} \) as a random quantity. The definitions (2) and (3) substantiate the first upshot.

The second upshot follows from the split of the density of total turbulence energy per unit mass, \( K = \frac{1}{2} \langle \mathbf{v}'^2 \rangle \), for the turbulent media with \( \boldsymbol{\Omega}, \mathbf{M} \neq 0 \) as

\[ K = K^\Omega + K^0, \]

(5)

where

\[ K^\Omega = \frac{1}{2} \mathbf{M} \cdot \boldsymbol{\Omega}, \]

(6)

and

\[ K^0 = \frac{1}{2} \left( \{ \mathbf{v}' \times \mathbf{R}' \} \cdot \{ \mathbf{v}' \times \mathbf{k}' \} \right). \]

(7)

The split reflects the decomposition of turbulence into two constituents, one characterized by \( \boldsymbol{\Omega} \) and \( \mathbf{M} \) (the orientated constituent) and the other by \( K^0 \) (the non-orientated constituent).

Figure 2a illustrates the corresponding to (5) split of turbulence energy on the energy graph for the cascading energy transfer in turbulent media constituted by the RK conception. The levels of energy release and gain are connected by vertical arrows pointing to the levels of energy gain. Insofar as the RK conception considers the turbulence energy fed by the average flow through the orientated turbulence constituent and the CTM neglects the orientated constituent of turbulence considering the energy of the non-orientated constituent obtaining its energy immediately from the average flow, the RK conception and the CTM prove contradicting. The restriction of the competence of the CTM to the description of the non-orientated constituent of turbulence obtaining its energy from the orientated constituent of turbulence resolves the contradiction.
2.3. The MSD and a particularization of the micropolar properties of turbulent continua

The method of structural decomposition (MSD) is based on the decomposition of the applied averaging procedure to a sequence of averaging procedures [4, 15-19], determined as an inverse of multiple averaging. If applied to the fluctuating constituent of an arbitrary random quantity, the decomposition represents this quantity in a form of superposition of its different variability constituents while for arbitrary two quantities only their variability constituents of the same variability level may prove mutually correlative.

While applied to the velocity fluctuation $v'$, $K$ and $M$ the application of the MSD results in

$$v' = \sum_{n=1}^{N} v_n,$$

$$K = \sum_{n=1}^{N} K_n \quad \text{and} \quad M = \sum_{n=1}^{N} M_n,$$

where

$$K_n = \frac{1}{2} M_n \rho_n + \sum_{p=n}^{N} K_{np}.$$

In (8)-(10): $N$ determines the decomposition range; $K_n$, $M_n$ and $\rho_n$ denote the averaged by $\langle \rangle$ quantities defined as $M$ and $\rho$ in (2) and (3) for $v'$ replaced by $v_n$; $K_{np}$ determine the $K_n$ constituents of motion of the order decreasing with decreasing $p$. As far as, similar to (4), each pair of $M_n$ and $\rho_n$ determine the respective average motion scale $\ell_n$, the sums (8) and (9) can also be interpreted as the decompositions of $K$ and $M$ by motion scales $\ell_n$. Notice the additive nature of decomposition of $K$ and $M$. The decomposition procedure represented by (8)-(10) enables a substantial particularization of description of cascading processes in turbulent media predicted by the RK conception.

Consider in the following an example of application of the MSD particularizing the explained in subsection 2.2 properties of average turbulent continuum following from the representation of the probability density function $f(v,k)$, as $f(v,k) = f_1(v|k)f_2(k)$, where $f_1(v|k) = f(\mathbf{v}|k)/f_2(k)$ and $f_2(k) = \int f(\mathbf{v},k)dv$. Using $f_1(v|k)$ and $f_2(k)$ we shall have for $v'$ the presentation

$$v' = v^* + \tilde{v}^*,$$

where (and henceforth) the overbar denotes the averaging by $f_1(v|k)$. It is evident that $v^*$ is statistically independent from $\tilde{v}^*$ and $k$. The independence declares that $v^*$ does not contribute to $\rho$ and $M$. Therefore, instead of (2) and (3), we can write also

$$\rho = \langle \mathbf{v}^* \times \mathbf{k} \rangle \quad \text{and} \quad M = \langle \tilde{\mathbf{v}}^* \times \mathbf{R} \rangle.$$

The expressions (12) particularize $\rho$ and $M$ as induced by the velocity fluctuation constituent $\tilde{v}^*$ only. In the energetic terms the particularization results in

$$K = K_1 + K_2,$$

$$K_2 = K^0 + K_2^0,$$

where

$$K_1 = \frac{1}{2} \langle v^*^2 \rangle, \quad K_2 = \frac{1}{2} \langle \tilde{v}^2 \rangle \quad \text{and} \quad K_2^0 = \frac{1}{2} \langle (\tilde{\mathbf{v}}^* \times \mathbf{R}) \cdot (\tilde{\mathbf{v}}^* \times \mathbf{k}) \rangle,$$

while

$$K_1^0 + K_2^0 = K^0.$$

The energy graph in Figure 3b illustrates the situation represented by (5), (13)–(15) as a particularization of the energy situation in Figure 3a. The CTM excluding the velocity fluctuation constituent $\tilde{v}^*$, results in

$$\rho = M = K_2 = K_2^0 \equiv 0 \quad \text{and} \quad K = K_1 = K^0.$$

3. THE FORMULATIONS OF THE TM
3.1. The RAT theory
3.1.1. The setup
The theory of rotationally anisotropic turbulence (the RAT theory) [4, 16, 20] exploits (2) and (3) to subject the formulation of the TM to the same type of set of the balance equation as applied in the theory of MF [13, 14], though complemented by the balance equation of energy characterized by $K^0$ and with the specifics of turbulent flows of incessant restoration of their eddy structure accounted for through the term of body moment in the balance equation for $M$. Expressions (3) and (7) suggest also algorithms for deriving the required balance equations from the equations of the CFM providing all terms of the derived equations with specific expressions through the momentum flow field characteristics. The RAT theory realizes this option by solving the emerging closure problem in agreement with the methodology applied in the theory of MF. As a result of the applied closure the turbulence viscosity properties are characterized besides the turbulent shear viscosity (associated with the symmetric constituent of the turbulent stress tensor) also by the turbulence rotational viscosity associated with the antisymmetric constituent of the turbulent stress tensor reflecting the interaction of the average flow with the orientated constituent of turbulence. The particularization of the presentation of the turbulence flow field properties explained in Section 2.3 adjusts the sense of antisymmetric constituent of the turbulent stress as reflecting the average effect of the flux of momentum $\rho w^\nu$ at a flow field point in the direction of the momentary curvature vectors of $\bar{v}^\nu$ streamlines either increasing or decreasing $M$. The explanation brings forth a substantial difference between the role of the velocity fluctuation constituent $\bar{v}^\nu$ playing in (12) and in the specification of turbulent stresses. In the latter case $\bar{v}^\nu$ participates in the combination $(\nu) + \bar{v}^\nu$ not contributing to the correlation moments of the velocity fluctuation components. Finally, besides introducing an extensive particularization into the treatment of dynamic, kinematic and energetic processes in turbulent media, the setup of the RAT theory includes as well a substantial particularization of descriptions of interaction of the flow field with external fields and of transport processes in turbulent media.

### 3.1.2. The converging effect of the RAT theory

In addition to comprising the situation declared by the RK conception and providing the physical background to the ambitions of application of methods of description of MF to the description of turbulent flows [27-29], the RAT theory also includes the CTM, specialized within this theory to the description of the turbulence constituent characterized by $K^0$. The interpretation of the turbulence constituents characterized by $K^0$ and by $\Omega$ and $M$ as the small-scale and the large-scale turbulence constituents, respectively, relates the RAT theory also to the LES modelling of turbulence [30]. In addition, the applied in the setup of the RAT theory KGP relates the RAT theory as well to the structure-based turbulence models [31] also aspiring to account for the properties of non-local turbulence structure through the local flow field characteristics. Finally, due to the equation of moment of momentum included in the turbulent motion description setup the RAT theory relates also to G. Mattioli [32], which first attempted to found this step, thought on the basis of semi-empirical turbulence treatment, and to [33, 34] establishing the correspondence between the average turbulence description and the theory of MF applying spatial averaging. In the latter cases the correspondence follows not from the physical reasons but from the finite size of the so-called elementary macro-volume.

### 3.1.3. Applications of the RAT theory

The applications of the RAT theory to the description of steady flows in round tubes, plane channels, between rotating cylinders and oscillating flows in round tubes [4, 16, 20], to the description of magneto-hydrodynamic flows in plane channels for small magnetic Reynolds number values [4, 16, 35], and to several problems of environmental fluid mechanics (the description of the velocity field in oscillating boundary layers [46]; the generalization of the Ekman layer to account for the Stokes drift and stratification effects [37, 38]; the explanation of the Gibraltar Salinity Anomaly [39] as an effect of anomalous turbulent diffusion perpendicular to the average salinity gradient; the descriptions of vertical transport of heat in the upper ocean [40, 41], of suspended sediments in open channels [42] and in the bottom layer of a natural water body [43]; the explanation of an increased net transport of the Antarctic Circumpolar Current as the turbulence effect [44, 45]; the modelling of formation of zonal winds in planetary atmospheres [46]; the proposal of an explanation of eddy-to-mean energy transfer in the Gulf Stream eliminating the necessity to apply negative value of the viscosity coefficient and/or the models of 2D turbulence [47]; modelling of eddy-driven flows over varying bottom topography [48]) have demonstrated, in addition to wide application perspectives of the RAT theory, a good agreement with the respective data.

We conclude the review of applications of the RAT theory by the following notes.

Firstly, all listed applications exploit the same set of the equations without any supplementary information except the included case-specific initial and boundary conditions. The situation characterizes the formulated the RAT theory as universal.
Secondly, the most of the listed applications adopt the simplification following from the RK conception – instead of all motion scales only the orientated (large-scale) constituent responsible for the interaction of turbulence with the average flow is accounted for. Due to the adopted simplification the most studied situations have been considered in terms of linear differential equations allowing for analytical presentation of the study results. Thirdly, the demonstrated agreement of the realized applications adopting the simplification with the respective data supports rather the RK conception than the CTM.

3.2. PMTF

PMTF [4, 16-19] complements the setup of the TM by the MSD. The complementation enables a considerable particularization of description of cascading processes in turbulent media in average terms. In particular, it evidences about the energy exchange within $K_n$ taking place between their energetic sublevels $K_{np}$ with coinciding $p$ characterizing the motions of the same degree of order. An essential property of the realized decomposition procedure is its invariance in respect to the decomposition range. So, in order to merge several adjacent energy constituents in (9) into a single constituent it would be necessary just to sum up the corresponding balance equations for $K^*$ resulting in a single equation having the same structure as the equations under summation. It is obvious that the closure of the deduced sets of balance equations for different situations of decomposition should not violate this invariance property. The restriction of invariance connects the values of medium coefficients of the same type, though reflecting the situations corresponding to different decomposition ranges.

4. CONCLUSIONS

The discussed PDT with its outcomes in the form of the RAT theory and the PMTF formulates an alternative to the dominating at present other two turbulence doctrines mentioned in Section 2 (Introduction). It is grounded on the RK turbulence conception with the formalism built up on two axiomatic principles and on one method enabling the realization of formalism of the PDT within the classical principles of statistical physics (mechanics) and continuum mechanics. The PDT esteems the RK conception due to its rich physical context as well as due to its deep historical roots going back to the physics of Lucrecé [4, 49] estimated by I. Prigogine [50] as a very first systematic portrayal of turbulence ever. The latter was interpreted as ascribing the RK conception with the sense expressing the most natural viewpoint to the turbulent form of motion through ages. Due to the enhanced physical background the PDT reveals several shortcomings in the wide-spread practice of discussion of turbulence problems. In particular, it stresses the need to determine the conditions of formation of properties of probability distribution (which have been so far left out of the setup of the SFM) as of an inseparable component of any statistical turbulence description. Moreover, besides declaring these conditions essential the PDT also specifies the terms in which these conditions should be formulated. In addition, specifying the properties of turbulence (expressed by the type of organization formed in specific conditions) the PDT evidences about the insufficient competence of the CFM for the description of turbulence. On the level of formulation of the TM the PDT also removes from the turbulent stress tensor the restriction of “evident symmetry” for long considered the most firmly established statement of any specification of formulation of the TM.

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6. REFERENCES