CLASSIFICATION OF CYCLIC SURFACES AND GEOMETRICAL RESEARCH OF CANAL SURFACES

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ABSTRACT
This review article is devoted to an analysis of the literature on the geometric researches of cyclic surfaces with generating circles of constant and various diameters. The shells with cyclic middle surfaces are particularly useful as connecting parts of pipelines, in spiral chambers of turbines in hydroelectric power stations, in public and commercial buildings, for example, as coverings of stadiums, in water attractions, and so on. This review article contains 45 references, and these are practically all original sources dealing with geometry of canal surfaces and with classification of cyclic surfaces.

Keywords: Cyclic surfaces, Canal surfaces, Differential geometry, Computer graphics, Classification of surfaces, Geometrical computer aided design.

1. INTRODUCTION
The investigation of the class of cyclic surfaces started from the research of tubular surfaces of constant diameter having straight or curvilinear axes. Further, many subclasses and types of cyclic surfaces were discovered and examined. Some cyclic surfaces have been named in honor of the geometers who presented these surfaces for the applications. For example, one may mention Joachimsthal’s surface, Dupin’s cyclides, or the surface of Virich. It should be noted that surfaces of revolution are the cyclic surfaces with straight axis but they are singled out into a special class of Surfaces of Revolution, which is why these surfaces will not be presented in this review.

The authors made a careful study of all available scientific and technical books and papers on the geometrical investigations of cyclic non-degenerating surfaces at issue and summarized the basic results.

2. TERMINOLOGY AND CLASSIFICATION OF CYCLIC SURFACES
The cyclic surface is formed by movement of a circle of variable or constant radius under any law in a three-dimensional space (Figure 1).

The equation of a cyclic surface in the vector form may be written as

\[ r = r(u, v) = \rho(u) + R(u)e(u, v), \]

where \( r(u, v) \) is the radius-vector of a cyclic surface; \( \rho(u) \) is the radius-vector of the directrix, i.e. of the line of the centers of generating circles; \( R(u) \) is the law of change of radius of circular generatrixes; \( e(u, v) \) is a vector-function of a circle of a unit radius in the plane of a generating circle with a normal \( \mathbf{n}(u) \) (Fig. 2); \( e_0(u), g_0(u) \) are the unit vectors of Cartesian coordinates in the plane of a generating circle; \( v \) is a polar angle in the plane of a circular generatrix.
Figure 1. The formation of cyclic surfaces

\[ e(u,v) = \cos v e_0(u) + \sin v g_0(u) \]
\[ g(u,v) = -\sin v e_0(u) + \cos v g_0(u) \]
\[ e_0(u) \times g_0(u) = e(u,v) \times g(u,v) = n(u) \]

Figure 2. A circle of unit radius

Coefficients of the first fundamental form \( E, F, G \) and the second fundamental form \( L, M, N \) are obtained with the help of formulas of differential geometry [1]:

\[
E = s^2 + 2s [ (te)R' + (tg)R(e_0'g_0) - R(tm)(en') ] + R^2 \left[ (e_0'g_0')^2 + (en')^2 \right]
\]
\[
G = R^2, \quad F = R \left[ s(tg) + R(e_0'g_0) \right], \quad \sigma = \left[ s(te) + R \right] / \left[ s(tm) - R(en') \right]
\]
\[
L = \left[ (te)s + R \right] / \sigma, \quad M = R \left[ (tm)s - R(en') \right] / \sigma
\]
\[
T_1 = s'(tm) + s^2 k(nv) + 2R' (en') - R [(en')^2 + 2(e_0'g_0')(gn')] R''
\]
\[
T_2 = s'(te) + s^2 k(ev) - R[(e_0'g_0')^2 + (en')^2] + R''
\]
\[
M = R \left[ (tm)s - (en')R(e_0'g_0) - (te)s + R'(gn')R' \right] / \sigma
\]
where \( t = \frac{\rho'}{s}; t, v \) are the unit vectors of a tangent and a normal to the line of the centers \( \rho \ (u) \).

Knowing the values of Gaussian quantities, one may calculate all geometrical parameters of a surface, i.e. the area of a surface fragment, lengths of the curves lying on the surface, curvatures of curves on the surface, and the Gaussian and mean curvatures of a surface. In some works, cyclic surfaces are given as enveloping surfaces of a single-parametrical family of spheres [2–4].

Classification of cyclic surfaces presented by S.N. Krivoshapko and V.N. Ivanov [5] includes both well-known groups, and the cyclic surfaces known to a narrow circle of geometricians (Figure 3). Some cyclic surfaces not entered into the classification should take a place in corresponding empty cells. Some cyclic surfaces simultaneously belong to several classes. For example, a subgroup of cyclic surfaces, \textit{Surfaces of Revolution}, may be isolated into in the individual class of the same name.

Any circle in the space may be described by a vector with the beginning coinciding with the center of the circle and with the direction along the normal to a plane in which the circle lies. The length of the normal vector is equal to radius of that circle, and this vector is called the clarifying or defining vector of the circle. Thus, any cyclic surface can be correlated in space with the ruled surface formed by a movement of the clarifying vector of the circle. As previously mentioned, the ruled surface is called a basic surface or a base of the cyclic surface. The beginnings of clarifying vectors set a line of the centers on the basic surface, and the ends of vectors define a line of radiuses.

3. THE SHORT DESCRIPTION OF TWO SUBCLASSES OF CYCLIC SURFACES

Apparently from the classification, the class «Cyclic Surfaces» includes 6 subclasses. Each subclass can be divided further into several groups of surfaces. The same group of surfaces can sometimes enter into different subclasses. For example, a group «Surfaces of Revolution» is included simultaneously into subclasses of «Canal Surfaces» and
«Normal Cyclic Surfaces», and a group «Dupin’s Cyclides» is a part of subclass of «Canal Surfaces» or «Cyclic Surfaces with Circles in Planes of the Pencil». Tubular surfaces also simultaneously enter into two subclasses.

3.1. A circular helical surface with a circular generatrix lying in the adjoining plane of a helical line of the centers of the circles

The defining vector of a circular generatrix is directed on a binormal of a helix of the centers of generating circles [7]. The parametrical equations of a helix of the centers of generating circles can be written as

\[ x = x(\vartheta) = a \cos \vartheta, \quad y = y(\vartheta) = a \sin \vartheta, \quad z = z(\vartheta) = pv. \]

The parametrical equations of the surface (Figure 4) can be rewritten in the following form

\[
\begin{align*}
x &= x(\vartheta, \nu) = (a + r \cos \vartheta) \cos \nu - r \sin \vartheta \sin \beta \sin \nu, \\
y &= y(\vartheta, \nu) = (a + r \cos \vartheta) \sin \nu + r \sin \vartheta \sin \beta \cos \nu, \\
z &= z(\vartheta, \nu) = pv - r \sin \vartheta \cos \beta,
\end{align*}
\]

where \( \vartheta \) is a central angle of the generating circle, \( 0 \leq \vartheta \leq 2\pi \); \( 0 \leq \nu \leq 2\pi \); \( r \) is the radius of the generating circle; \( \beta \) is the angle between a binormal of a helix and a plane \( z = 0 \):

\[
\begin{align*}
tg \beta &= -\frac{a}{p}, \\
\sin \beta &= \frac{a}{\sqrt{p^2 + a^2}}, \\
\cos \beta &= -\frac{p}{\sqrt{p^2 + a^2}}, \\
\beta &= \frac{\pi}{2} + \arctg \frac{p}{a}.
\end{align*}
\]

\[ p = 0.5 \text{ m}; \quad a = r = 1 \text{ m} \]

Figure 4. A circular helical surface with a circular generatrix lying in the adjoining plane of a helical line of the centers of the circles.

The coefficients of fundamental forms of the surface are presented in [5]. Coordinate lines \( \vartheta \) coincide with the generating circles. The angle \( \chi \) between non-orthogonal non-conjugate curvilinear coordinates \( \nu \) and \( \vartheta \) is calculated under the formula:

\[
\cos \chi = \frac{\sqrt{a^2 + p^2} \left( \frac{r a}{a^2 + p^2} + \cos \vartheta \right)}{\sqrt{r^2 a^2 \sin^2 \vartheta + (a + r \cos \vartheta)^2 + p^2}} \neq 0.
\]

3.2. Canal surfaces

There are disagreements in certain literature in definitions of canal surfaces. V.I. Shulikovsky [2] supposed that one family of lines of principal curvatures of canal surfaces consists of circles. This surface is formed by a single-parameter family of spheres. This definition was seconded by A.P. Norden [3] and V.N. Ivanov [8].

But V.F. Kagan [9] insisted that canal surfaces are formed by movement of a circle of variable radius and these circles lie in the normal planes of the line of centers of these circles. The Kagan’s definition was used in works [10, 11]. So, if one uses the Kagan’s definition then he must decline a conclusion that generating circles of canal surfaces are the lines of principle coordinates of normal cyclic surface [12-14].
Besides, it is possible to put one more argument against the definition of canal surfaces, accepted in works [9-11]. In geometrical literature, Dupin’s cyclides i.e. surfaces with two families of the circles which are lines of principal curvatures are called two-canal surfaces [15–17]. It is easy to prove that planes of circular generatrices of Dupin’s cyclides don’t coincide with normal planes of centerlines of families of the circles [18, 19].

The authors in their review use the definition given by V.I. Shulikovsky [2].

If generating circles of cyclic surface i.e. the coordinate lines \( u = \text{const} \) are lines of principal curvatures then a condition

\[
MG - NF = 0
\]

must be fulfilled. In a condition (2), \( G, F, M, N \) are coefficients of fundamental forms of a surface (1).

We have four special types of canal surfaces:

1. **Tubular surfaces** envelope one-parametrical family of spheres of constant radius. Circular lines of principal curvature lie in normal planes of the center line.
2. **Dupin’s cyclides** are two-canal surfaces and both families of lines of principle curvatures of cyclides consist of circles.
3. **Canal surface of Joachimsthal** is a canal surface with a flat arbitrary curve as the center line and with circular lines of principle curvatures lying in the planes of one pencil.
4. **Surfaces of revolution** have a straight line of centers of generating spheres and this line is the axis of revolution of the proper meridian.

### 3.2.1. Tubular surfaces

Tubular surfaces are described by the equation (1), but it is necessary to put that

\[
R(u) = R = \text{const}, \quad dR(u)/du = 0,
\]

and then a vector equation of tubular surfaces may be presented in the following form

\[
r = r(u,v) = \rho(u) + Re(u,v).
\]

The expressions for coefficients of the first fundamental form \( E, F, G \) and the second fundamental form \( L, M, N \) are simplified and become

\[
E = A^2 = s^2 \left[ 1 - kR \cos(v + \theta) \right]^2, \quad F = 0, \quad G = B^2 = R^2, \quad \text{Where} \quad \omega = v + \theta;
\]

\[
\theta = \theta(u) = -\int k\omega du + \theta_0;
\]

\[
L = -s^2 k \left[ 1 - Rk \cos(v + \theta) \right] \cos(v + \theta), \quad M = 0, \quad N = R, \quad k_1 = \frac{k \cos(\theta + v)}{1 - kR \cos(v + \theta)}, \quad k_2 = \frac{1}{R},
\]

where \( k, \kappa \) = curvature and torsion of a centerline. Thus, circular generatrices in tubular surfaces coincide with one family of lines of principal curvature and the coordinate network \( u, v \) is a network of line of principal curvatures. The vector of a normal of tubular surfaces lies in a plane of the generating circles which are geodesic lines. It is possible to consider tubular surfaces both cyclic surface and Monge’s surface of double curvature.

If the tubular surface has a flat centerline, \( \theta = 0 \) as \( \kappa = 0 \), then coefficients of the fundamental forms of surface and its principle curvatures can be expressed as

\[
E = A^2 = s^2 \left[ 1 - kR \cos v \right]^2, \quad F = 0, \quad G = B^2 = R^2,
\]

\[
L = -s^2 k \left[ 1 - Rk \cos v \right] \cos v, \quad M = 0, \quad N = R, \quad k_1 = \frac{k \cos v}{1 - kR \cos v}, \quad k_2 = \frac{1}{R},
\]

where \( k \) = curvature of the flat centerline.
All tubular surfaces shown in Figure 5 are presented in encyclopedia [6] where their parametrical equations, coefficients of the fundamental forms of surface, and the value of principle curvatures are given also.

The additional information about the tubular surfaces shown in Fig. 4 can be found in [20-24]. Yu.Z. Shvidenko and L.S. Panasjuk [25] have offered a method of approximation of a tubular helical surface with the help of continuous winding of ribbon of periodically changing width. Similar problems were solved in works [26, 27].
3.2.2. Dupin’s cyclides

Throughout the last two centuries many scientists, including J.C. Maxwell [16], A. Caley [17], Frank Hubert [28], Meszrios Ferenc [29], V.I. Milinsky [30], V.I. Shulikovsky [2], V.F. Kagan [9], A.M. Yakubovsky [31, 32], I.K. Boikov [33] were engaged in research of various aspects of geometry of these surfaces. In their works, drawings and photos of breadboard models of these surfaces are shown, various forms of the equations and geometrical characteristics of these surfaces are presented, and classification of surfaces is offered. Having been double canal surfaces, Dupin’s cyclides are enveloping surfaces of two one-parametrical families of spheres. On this basis, the implicit and parametrical equations of Dupin’s cyclides are received in works [2, 31-33]. In works [31, 32], it is shown that Dupin’s cyclides are canal surfaces of Joachimsthal, i.e. circular generatrixes of each family of circles of these surfaces lie in planes of the pencil.

However, for the description of geometry in these works, the traditional implicit and parametrical equations of enveloping surfaces of the families of spheres are used. These equations do not allow revealing character of formation of surfaces in shape of cyclides of Dupin without special studying.

Having studied Dupin’s cyclides as canal surfaces of Joachimsthal, character of formation of these surfaces becomes clear. The planes rotating about an axis passing through the pole are drawn from the general pole (Figure 6). The generating circle is drawn in each plane. Position of the center and radius of a generating circle should be defined so that these circles were lines of principal curvature of the surface in question. For receiving equation of Dupin’s cyclide, it is necessary to have the equation of a circle in polar system of coordinates with the beginning of coordinates in the pole. Thus the pole is displaced concerning the center of a directing circle (Figure 6a).
The equation of a directing circle can be written in the following form

\[ r_2(\alpha) = a \cdot \bar{r}_2(\alpha); \quad \bar{r}_2(\alpha) = \gamma \cdot \cos \alpha + \sqrt{\mu^2 + \gamma^2 \cdot \sin^2 \alpha}. \]  

(3)

where \( a \mu \) is radius of a directing circle; \( a \gamma \) is an eccentricity of the pole relative to the center of a directing circle.

Dupin’s cyclide the directing curve of which is a circle, is called Dupin’s cyclide of the fourth order (Figure 7) because its equation can be written in an implicit form with the help of the 4th order algebraic equation [33, 34]

\[ (x^2 + y^2 + z^2 - \mu^2 + b^2)^2 = 4(a \gamma - c \mu)^2 + 4b^2 y^2, \]

where \( \mu, b, c^2 = a^2 - b^2, a \) are the constants entering into the parametrical equations of a focal ellipse and a hyperbola [35]. As it was noted earlier canal surfaces can be formed as enveloping surfaces of the family of spheres with the centers in the points of the directing curve.

![Figure 7. Dupin’s Cyclides of the 4th order [18]](image_url)
In a limit when the radius of a directing circle aspires to the infinity, the directing circle turns into a straight line but a canal surface of Joachimsthal turns into Dupin’s cyclide of the third order (Figure 8). The focal parabolas of the third order Dupin’s cyclides located in two mutually perpendicular planes. The centers of the spheres forming the cyclides in question lie on these focal parabolic curves.

Dupin’s cyclide of the third type of the fourth order is circular torus.

The influence of constant parameters entering into equations of Dupin’s cyclides on the forms of cyclides was studied in [18, 33].

Except the considered constructive-mathematical method of design and designing of cyclides as enveloping surface of family of spheres [33, 35], researches on design of Dupin’s cyclides from congruence of circles are presented in [32]. The additional materials with the results devoting to geometrical researches of Dupin’s cyclides are given in [36–39].

3.2.3. Canal surfaces of Joachimsthal

Joachimsthal’s surface is a surface with a family of plane lines of curvature lying in the planes of the pencil [8]. Canal surface is called a cyclic surface with a family of circles \( R(u) \), being lines of curvature of a surface. If circles of canal surfaces lay in the planes of pencil, the surface is canal surface of Joachimsthal. A centerline of canal surfaces of Joachimsthal is a flat curve \( r(u) \), therefore canal surfaces of Joachimsthal are included into a group of cyclic surfaces with circles in planes of a pencil and with a flat centerline.

The cyclic surface with circles in planes of a pencil and with a flat line of the centers may be a canal surface if a condition

\[
\left[ r^2(u) - R^2(u) \right] = 0, \quad r^2(u) - R^2(u) = [r(u) - R(u)] \cdot [r(u) + R(u)] = r_1(u) \cdot r_2(u) = \pm c^2
\]  

(4)
is satisfied, where \( c = \text{const} \) (Figure 9). Using this condition one can get three ways of forming canal surfaces of Joachimsthal [40]:

I) the surface is formed by rotation of a circle of variable radius with reservation of constant distance from the pole to the point of contact with generating circle. This distance is equal to \( c \), i.e. \( r(u) > R(u), \ r'(u) - R'(u) = +c^2 \) (Fig. 9a);

II) the surface is formed by rotation of a circle of variable radius around the general chord with a length equal \( 2c \), then \( r(u) < R(u), \ r'(u) - R'(u) = -c^2 \) (Figure 9b);

III) the surface is formed by rotation of a circle of variable radius around the general tangent. In that case \( r(u) = R(u), \ r_1(u)=0, \ r_2(u)=2R(u), \ c = 0 \) (Figure 9c).

Thus, for forming of canal surfaces of Joachimsthal it is enough to take any flat centerline \( r(u) \) of generating circles relative to the pole and a parameter \( \pm c^2 \). The radius of circular generatrixes should be defined from a condition (4).

Post-graduate student of a chair of strength of materials of the People’s Friendship University of Russia N.J. Abbushi [41] has constructed gyps models of canal surfaces of Joachimsthal explaining three ways of their formation (Figure 10).
3.2.3.1. Epitrochoidal surface

The $M$ point located on a plane of a circle with the $a$ radius, which rolls without sliding on other motionless circle with $b$ radius, forms an epitrochoidal line. The planes of these two circles constitute a constant corner $\gamma$. The distance from a point of $M$ to the center of a mobile circle is equal to $\mu a$ ($\mu = 1$, or $\mu < 1$, or $\mu > 1$). Changing parameter $\gamma$ from 0 to $2\pi$, it is possible to receive a family of epitrochoidal curves which will form epitrochoidal surface (Figure 11) [6]. Surface $\Phi$ envelops a system of spherical surfaces and touches with them along the circles. The theorem of Joachimsthal proves that the family of circles of epitrochoidal surfaces is lines of curvature; hence, a surface $\Phi$ is a special case of canal surface of Joachimsthal.

Figure 10. The gyps models of canal surfaces of Joachimsthal

![Figure 10](image)

Figure 11. The epitrochoidal surfaces in lines of principal curvatures

![Figure 11](image)
The implicit equation of epitrochoidal surface can be presented in the form [42]:

\[(x^2 + y^2 + z^2 - 2\mu a)^2 = 4\alpha^2(x^2 + y^2)\,.
\]

The implicit equation of epitrochoidal surface is obtained by V.G. Steblyanko for a case with \(a = b\) [42, 43]. The \(xOy\) plane crosses epitrochoidal surface along epitrochoid which also is named limaçon of Pascal.

An epitrochoidal surface maybe defined by the parametrical equations:

\[x = x(\alpha, v) = 2R(\alpha)\cos^2 v \cos \alpha, \quad y = y(\alpha, v) = 2R(\alpha)\cos^2 v \sin \alpha, \quad z = z(\alpha, v) = R(\alpha)\sin 2v,\]

where \(R(\alpha) = a (1 + \mu \cos \alpha)\) is the radius of a generating circle, \(\alpha\) is a corner between an axis \(Ox\) and a plane of a circular generatrix; \(0 \leq \alpha \leq 2\pi\), \(v = \gamma/2\) is an angle between a radius-vector of a surface and a plane of a motionless circle; \(-\pi/2 \leq v \leq \pi/2\). At this way of the task, one recognizes that a surface generates by rotation of a mobile circle with radius \(a\) about its tangent in the point of a contact with a motionless circle with radius \(b = a\). Generating circles of the surface lie in a plane of one pencil. The beginning of coordinates is placed in a double conic point of the surface.

Having the parametrical equations of epitrochoidal surface one may derived

\[A^2 = 4\left[R^2 \cos^2 v + a^2 \mu^2 \sin^2 \alpha \right] \cos^2 v, \quad F = 2a\mu R \sin 2v \sin \alpha, \quad B^2 = 4R^2, \quad N = \frac{-4R^2}{\sqrt{R^2 + a^2 \mu^2 \sin^2 \alpha}}, \quad L = -2a\mu R \cos \alpha + R^2 \cos 2v + 2a^2 \mu^2 \sin^2 \alpha \cos v, \quad M = \frac{-2a\mu R \sin 2v \sin \alpha}{\sqrt{R^2 + a^2 \mu^2 \sin^2 \alpha}}.\]

A surface in question is covered by non-orthogonal \((F \neq 0)\) non-conjugate \((M \neq 0)\) system of curvilinear coordinates.

A vector equation of epitrochoidal surface in lines of principal curvatures may be written in the following form (Figure 11) [44]:

\[r = r(\alpha, \beta) = 2R(\alpha)[\cos \alpha i + \sin \alpha j + R(\beta)f(\beta)k]/D(\alpha, \beta),\]

where \(D(\alpha, \beta) = 1 + R^2(\alpha)f^2(\beta)/a^2; \ f(\beta)\) is any twice differentiated function, for example, \(f(\beta) = \tan \beta\).

3.2.3.2. Cyclic surface of Virich

Cyclic surface of Virich [45] recently opened is the closed surface with three planes of symmetry (Figure 12). The surface is generated by the circles of variable radius lying in the planes of a pencil and passing through the fixed straight line. The fixed straight line of a pencil of planes with circles passes through a point of intersection of three planes of symmetry perpendicularly to one plane of symmetry in which the flat centerline of generating circles is located. S.O. Virich offered the following parametrical equations of cyclic surface in question [45]:

\[x = x(t, v) = \frac{1}{2} \left[f(v)(1 + \cos t) + (d^2 - c^2) \frac{1 - \cos t}{f(v)} \right] \cos v,\]

\[y = y(t, v) = \frac{1}{2} \left[f(v)(1 + \cos t) + (d^2 - c^2) \frac{1 - \cos t}{f(v)} \right] \sin v,\]

\[z = z(t, v) = \frac{1}{2} \left[f(v) - \frac{d^2 - c^2}{f(v)} \right] \sin t,\]

where \(f(v) = \frac{ab}{\sqrt{a^2 \sin^2 v + b^2 \cos^2 v}}, 0 \leq t \leq 2\pi; 0 \leq v \leq 2\pi; \ a, b, c, d\) are constants.
In Figure 12, cyclic surfaces of Virich and their fragments are represented at different values of constants $c$ and $d$.

Cyclic surface of Virich may be described by a vector equation:

$$ r = r(t, v) = [R(t,v)h(v) + \psi(v)k]/2, $$

where $h(v) = i \cos v + j \sin v; \quad p = d^2 - c^2; \quad R(t,v) = \varphi(v) + \psi(v) \cos t; \quad \varphi = \varphi(v) = f(v) + p f(v);\quad \psi = \psi(v) = f(v) - p f(v)$.

Coefficients of the fundamental forms of surface maybe derived as:

$$ A^2 = [(\varphi + \psi \cos t)^2 + \varphi'^2 + \psi'^2 + 2 \varphi' \psi' \cos t]/4, \quad F = -\varphi' \psi \sin t/4; \quad B^2 = \psi^2/4; $$

$$ L = [(\varphi + \psi \cos t) [\varphi'' + \psi'' - (\varphi + \psi \cos t)] \cos t + + 2(\varphi' + \psi' \cos t)(\psi' + \varphi' \cos t)]/(2 \sigma); $$

$$ M = \psi(\psi' + \varphi' \cos t) \sin t/(2 \sigma); \quad N = -\psi(\varphi + \psi \cos t)/(2 \sigma), $$

where $\sigma^2 = (\varphi + \psi \cos t)^2 + (\psi' + \varphi' \cos t)^2$. 

Fig. 12. Cyclic surfaces of Virich
Coordinate lines \( v (t = \text{const}) \) coincide with generating circles and form one family of lines of principal curvatures. Hence, the cyclic surface of Virich can be ranked to canal surfaces of Joachimsthal. Coordinate lines \( t (v = \text{const}) \) aren’t lines of principal curvatures.

4. CONCLUSION

There are many interesting results in scientific works devoted to geometrical investigation of cyclic surfaces especially of tubular surfaces. But the authors did not find any review on this theme and decided to gather materials having begun from classification of cyclic surfaces and from geometrical research of canal surfaces. Canal surfaces were chosen because this subclass contains well-known groups of cyclic surfaces.

The authors hope that the literature resulting from the review will help geometerics and post-graduate students choose theme of scientific research.

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