BAYESIAN ESTIMATION OF CHANGE POINT IN AUTOREGRESSIVE PROCESS

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ABSTRACT

Let AR(1) model be \( X_i = \beta_1 X_{i-1} + \varepsilon_i \) where \( \varepsilon_i \) are assumed to have Normal distribution with mean 0 and variance \( \sigma_i^2 \) and autoregression coefficient \( \beta_1 \) but later it was found that there was a change in the system at some point of time \( m \) and it is reflected in the sequence after \( X_m \) by change in variance of error term \( \sigma_i^2 \) and autoregression coefficient \( \beta_2 \). The problem of study is: When and where this change has started occurring. This is called change point inference problem. The estimators of \( m, \beta_1, \) and \( \beta_2 \) are derived under asymmetric loss functions namely Linex loss & General Entropy loss functions. Both the non-informative and informative prior are considered. The effects of prior consideration on Bayes estimates of change point are also studied.

Keywords: Bayes estimates; change point; Normal distribution; Auto regressive process.

1. INTRODUCTION

In this paper, we have done Bayesian analysis of a first order autoregressive process subject to one change in both the variance of the error terms and the autoregression coefficients at an unknown time point. The main emphasis is to derive the posterior distributions of the change point and the autoregression parameter.

The change point problem has been considered by many authors from various viewpoints. From a Bayesian viewpoint, primary interest in the study of structural change has focused on models assuming independence between observations. In practice, this assumption may be violated as in the case of an autoregressive time series. In addition, most studies have been concerned with the problem of a change in the mean of a sequence of independent random variables. These are many contributions to this area since introduced by Page (1954). The Monograph of Broemeling and Tsurumi (1987) on structural changes, P.N. Jani & Mayuri Pandya (1999), Ebrahim and Ghosh (2001) and a survey by Zacks (1983), Pandya, M. and Jani, P. N. (2006), Pandya, M. and Bhatt, S. (2007), Mayuri Pandya & Prabha Jadav (2008), Pandya, M. and Jadav, P. (2010), and Pandya, M. and Bhatt, S. (2011) are useful references. From a Bayesian viewpoint, Chernoff and Zacks (1964) and Kander and Zacks (1966) derived Bayesian tests for detecting a change in the mean of a sequence of independent normal random variables.

It may happen that at some point of time instability in the sequence of observation is obtained. The problem of study is: When and where this change has started occurring. This is called change point inference problem. Bayesian ideas may play an important role in the study of such change point problem and has been often proposed as a valid alternative to classical estimation procedure.

While many Bayesian studies assume homoscedasticity in the error terms, only a few studies have investigated the problem of a change in the variance at an unknown time point (e.g. Smith, 1975). He gave posterior probabilities for the time point when a variance change occurs. Using sampling theory, Hsu (1977) derived two tests for a variance shift in a sequence of independent random variables. For a sequence of independent and normally distributed random variables, Menzefricke (1981) obtained the posterior distributions of both change point and variance.

Abraham and Wei (1984) derived the posterior distributions of the parameters of a time series assuming a known inflation or deflation of the errors variance. Assuming the change point known, Bromeling and Tsurumi (1987) considered a linear model with correlated error terms and where the variance and the correlation coefficient change. Tsay (1988) proposed a procedure to detect outliers, level shifts, and variance changes in an autoregressive moving average model. Inclan and Tiao (1994) considered the problem of multiple change points in the variance of a sequence of independent observations.

In section 2, we have developed a change point model related to AR (1) with change in errors variance. In section 3, we obtained posterior densities of \( \beta_1, \beta_2 \) and \( m \) for this model. We derive Bayes estimators of \( \beta_1, \beta_2 \) and \( m \) under symmetric loss functions and asymmetric loss functions in section 4. In section 5, we have presented a numerical study to illustrate the above technique on generated observations. In section 6, we have studied the sensitivity of the Bayes estimates with respect to change in prior of the parameters. We also studied of the effects of different loss functions on Bayes estimates. In this study, we have done simulation study by generating 10,000 different random samples and it is described in section 7. Section 8 shows a detailed conclusion.
2. PROPOSED AR(1) MODEL

Assuming a change in both the autocorrelation coefficient of an AR(1) process and the variance of the error terms at an unknown time point \( m \), the model is given by:

\[
X_i = \begin{cases} 
\beta_1 x_{i-1} + \epsilon_i, & i = 1, 2, ..., m, \\
\beta_2 x_{i-1} + \epsilon_i, & i = m + 1, ..., n. 
\end{cases}
\]  

(2.1)

Where, \( \beta_1 \) and \( \beta_2 \) are unknown autocorrelation coefficients, \( x_i \) is the \( i \)th observation of the dependent variable, the error terms \( \epsilon_i \) are independent random variables and follow a \( N(0, \sigma^2) \) for \( i=1,2,\ldots,m \) and a \( N(0, \sigma_2^2) \) for \( i=m+1, \ldots,n \), where \( \sigma^2 \) and \( \sigma_2^2 \) are known. \( m \) is the unknown change point and \( x_0 \) is the initial quantity.

The likelihood function is given by,

\[
L(\beta_1, \beta_2, m | X) = k_1 \exp \left[ -\frac{1}{2} \beta_1^2 (S_m^2 / \sigma^2) + \beta_1 (S_{m+2} / \sigma^2) - S_m / 2\sigma^2 \right] \sigma_2^{-m} \sigma_2^{-(n-m)}
\]

\[
\exp \left[ -\frac{1}{2} \beta_2^2 (S_m^2 / \sigma_2^2) + \beta_2 (S_{n+2} - S_n / \sigma_2^2) - (S_m - S_{n+2} / 2\sigma_2^2) \right]
\]

(2.2)

Where,

\[
S_k = \sum_{i=k+1}^{k+m} x_i
\]

\[
k_1 = (2\pi)^{-n/2}
\]

(2.3)

3. BAYES ESTIMATION

The ML methods, as well as other classical approaches are based only on the empirical information provided by the data. However, when there is some technical knowledge on the parameters of the distribution available, a Bayes procedure seems to an attractive inferential method. The Bayes procedure is based on a posterior density, say, \( g(\beta_1, \beta_2, m | X) \), which is proportional to the product of the likelihood function

\[ L(\beta_1, \beta_2, m | X) \]

with a prior joint density, say, \( g(\beta_1, \beta_2, m) \) representing uncertainty on the parameters values.

\[
g_1(\beta_1, \beta_2, m | X) = \frac{L(\beta_1, \beta_2, m | X) \cdot g(\beta_1, \beta_2, m)}{\sum_{m=1}^{n-1} \int_{\beta_1} \int_{\beta_2} \int_0 L(\beta_1, \beta_2, m | X) \cdot g(\beta_1, \beta_2, m | X) \, d\beta_1 \, d\beta_2}
\]

3.1 Using Informative Priors For \( \beta_1 \)And \( \beta_2 \)

The prior distributions of the unknown parameters are assigned as follows: the prior distribution of \( \beta_1 \) is normal with mean \( \mu_1 \) and variance \( \sigma_1^2 \), \( i=1,2 \). Furthermore, we assume that \( \beta_1, \beta_2 \) and \( m \) are independent random variables.

As in Broemeling et al. (1987), we suppose the marginal prior distribution of \( m \) to be discrete uniform over the set \( \{1, 2, \ldots, n-1\} \).

\[
g_1(\beta_1) = \left(\sqrt{2\pi \sigma_1^2} \right)^{-1} \exp \left[ -\frac{1}{2\sigma_1^2} (\beta_1 - \mu_1)^2 \right] \quad -\infty < \beta_1 < \infty, \quad a_1, \mu_1 > 0
\]

\[
g_1(\beta_2) = \left(\sqrt{2\pi \sigma_2^2} \right)^{-1} \exp \left[ -\frac{1}{2\sigma_2^2} (\beta_2 - \mu_2)^2 \right] \quad -\infty < \beta_2 < \infty, \quad a_2, \mu_2 > 0
\]

\[
g_1(m) = \frac{1}{n-1}
\]

The joint prior distribution of the parameters \( \beta_1, \beta_2 \) and \( m \) is given by,

\[
g_1(\beta_1, \beta_2, m) = k_2 \exp \left[ -\frac{1}{2\sigma_1^2} (\beta_1^2 - 2\beta_1\mu_1 + \mu_1^2) \right] \exp \left[ -\frac{1}{2\sigma_2^2} (\beta_2^2 - 2\beta_2\mu_2 + \mu_2^2) \right]
\]

(3.1)

where,

\[
k_2 = (2\pi (n-1))^\frac{1}{2} (\sigma_1^{-2} \sigma_2^{-2})^{-1}
\]

(3.2)

By Bayes’ theorem, the joint posterior density of \( \beta_1, \beta_2 \) and \( m \) say \( g_1(\beta_1, \beta_2, m | X) \) is,

\[
g_1(\beta_1, \beta_2, m | X) = k_3 h^{-1}(X) \exp \left[ -\frac{1}{2} \beta_1^2 A_{1m} + \beta_1 B_{1m} \right] \exp \left[ -\frac{1}{2} \beta_2^2 A_{2m} + \beta_2 B_{2m} \right] \exp \left[ -\frac{1}{2} S_m (\sigma_1^{-2} \sigma_2^{-2}) \sigma_1^{-m} \sigma_2^{-(n-m)} \right]
\]

(3.3)

Where, \( \int L(\beta_1, \beta_2, m | X) \cdot g(\beta_1, \beta_2, m | X) \, d\beta_1 \, d\beta_2 \)

\[ h_1(X) \]

is the marginal density of \( X \) given by,

\[
h_1(X) = \sum_{m=1}^{n-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L(\beta_1, \beta_2, m | X) \cdot g(\beta_1, \beta_2, m | X) \, d\beta_1 \, d\beta_2
\]

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\[ k_3 \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \beta_1^2 A_{1m} + \beta_1 B_{1m} \right] \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \beta_2^2 A_{2m} + \beta_2 B_{2m} \right] d\beta_1 d\beta_2 \]

exp \left\{ -\frac{1}{2} \left\{ S_{m3} \left( \sigma_1^{-2} \sigma_2^{-2} \right) \right\} \sigma_1^{-m} \sigma_2^{-(n-m)} \right\} (3.4)

where,

\[
A_{1m} = \frac{S_{m1} + 1}{a_1}, \quad A_{1m} > 0, B_{1m} = \frac{S_{m2} - \mu_1}{a_1}, \\
A_{2m} = \frac{S_{m1} - S_{m3}}{a_2} + 1, \quad A_{2m} > 0, B_{2m} = \frac{S_{m2} - S_{m3}}{a_2} + \frac{\mu_2}{a_2},
\]

and the integrals,

\[
l_1(m) = \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \beta_1^2 A_{1m} + \beta_1 B_{1m} \right] d\beta_1
\]

\[
= \left[ \frac{\sqrt{2\pi} e^{\frac{\beta_1^2}{2A_{1m}}} \psi A_{1m}^{(a_1+1)/2} \Gamma\left(\frac{a_1+1}{2}\right)}{\sqrt{\pi}} \right]
\]

\[
l_2(m) = \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \beta_2^2 A_{2m} + \beta_2 B_{2m} \right] d\beta_2
\]

\[
= \left[ \frac{\sqrt{2\pi} e^{\frac{\beta_2^2}{2A_{2m}}} \psi A_{2m}^{(a_2+1)/2} \Gamma\left(\frac{a_2+1}{2}\right)}{\sqrt{\pi}} \right]
\]

Using (3.5) and (3.6) in (3.4), we get

\[ h_{1}(X) = k_3 \sum_{m=1}^{n-1} T_{1}(m). \]

Where,

\[ T_{1}(m) = l_1(m) l_2(m) \exp \left\{ -\frac{1}{2} \left\{ S_{m3} \left( \sigma_1^{-2} \sigma_2^{-2} \right) \right\} \sigma_1^{-m} \sigma_2^{-(n-m)} \right\} \]

\[
k_3 = k_1 k_2 \exp \left\{ -\frac{1}{2} \left[ \frac{\mu_1}{a_1} + \frac{\mu_2}{a_2} \right] \right\} e^{-\frac{\mu_1^2 + \mu_2^2}{2}}
\]

Integrating \( g(\beta_1, \beta_2, m | X) \) on \( (\beta_1, \beta_2) \) leads to the posterior distribution of change point \( m \). Marginal posterior density of change point \( m \) is given by,

\[ g_{1}(m | X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{1} (\beta_1, \beta_2, m | X) d\beta_1 d\beta_2 \]

\[ = T_{1}(m) / \sum_{m=1}^{n-1} T_{1}(m) \]

Where,

\[ T_{1}(m) \] as given in (3.8).

Marginal posterior densities of \( \beta_1 \), say \( g_{1}(\beta_1 | X) \) and of \( \beta_2 \), say \( g_{1}(\beta_2 | X) \) are given by,

\[ g_{1}(\beta_1 | X) = \sum_{m=1}^{n-1} g_{1} (\beta_1, \beta_2, m | X) d\beta_2 \]

\[ = k_3 \sum_{m=1}^{n-1} \exp \left\{ -\frac{1}{2} \beta_1^2 A_{1m} + \beta_1 B_{1m} \right\} I_{2}(m) \exp \left\{ -\frac{1}{2} \left\{ S_{m3} \left( \sigma_1^{-2} \sigma_2^{-2} \right) \right\} \sigma_1^{-m} \sigma_2^{-(n-m)} \right\} h_{1}^{-1}(X), \]

where,

\[ S_{m3}, I_{2}(m) \] and \( h_{1}^{-1}(X) \) are as given in (2.3)(3.6) and (3.4) respectively. \( A_{1m}, B_{1m} \) and \( k_3 \) are as given in (3.4)a and (3.9).

Marginal posterior density of \( \beta_2 \) is given by,

\[ g_{1}(\beta_2 | X) = \sum_{m=1}^{n-1} g_{1} (\beta_1, \beta_2, m | X) d\beta_1 \]

\[ = k_3 \sum_{m=1}^{n-1} \exp \left\{ -\frac{1}{2} \beta_2^2 A_{2m} + \beta_2 B_{2m} \right\} I_{1}(m) \exp \left\{ -\frac{1}{2} \left\{ S_{m3} \left( \sigma_1^{-2} \sigma_2^{-2} \right) \right\} \sigma_1^{-m} \sigma_2^{-(n-m)} \right\} h_{1}^{-1}(X), \]

where,

\[ S_{m3}, I_{1}(m) \] and \( h_{1}^{-1}(X) \) are as given in (2.3) (3.5) and (3.4) respectively. \( A_{2m}, B_{2m} \) and \( k_3 \) are as given in (3.4)a and (3.9).

### 3.2 Using Non-Informative Priors For \( \beta_1 \) And \( \beta_2 \)

A non-informative prior is a prior that reflects indifference to all values of the parameter, and adds no information to that contained in the empirical data. Thus, a Bayes inference based upon non-informative prior has generally a theoretical interest only, since, from an engineering viewpoint, the Bayes approach is very attractive for it allows incorporating expert opinion or technical knowledge in the estimation procedure. However, such a Bayes inference acquires large interest in solving prediction problems when it is extremely difficult, if not at all possible, to find a classical solution for the prediction problem, because classical prediction intervals are numerically equal to the
Bayes ones based on the non-informative prior density. Hence, the Bayes approach based on prior ignorance can be viewed as mathematical method for obtaining classical prediction intervals.

Let’s consider non-informative prior on $\beta_1$ and $\beta_2$ be standard normal and change point $m$ to be uniform.

Hence, the joint prior distribution of the parameters $\beta_1, \beta_2$ and $m$ is given by,

$$g_2(\beta_1, \beta_2, m) = (2\pi)^{n/2} \exp \left[ -\frac{1}{2} \beta_1^2 A_{3m} + \beta_1 B_{3m} \right]$$  

(3.13)

Using the likelihood function (2.2) with the joint prior density of $\beta_1$, $\beta_2$ and $m$, the joint posterior density of $\beta_1$, $\beta_2$ and $m$ say, $g_2(\beta_1, \beta_2, m|X)$ is obtained as,

$$g_2(\beta_1, \beta_2, m|X) = k_4 \exp \left[ -\frac{1}{2} \beta_1^2 A_{3m} + \beta_1 B_{3m} \right]$$  

(3.14)

where,

$$h_2(X)$$ is the marginal density of $X$ and is given by,

$$h_2(X) = \frac{m}{\sqrt{\pi}} \prod_{i=1}^{n} \left[ \frac{1}{\sqrt{A_{3m}}} \right]$$

(3.15)

Using (3.17) and (3.18) in (3.15), we get

$$h_2(X) = k_4 \sum_{m=1}^{n} T_2(m).$$

(3.19)

Where,

$$T_2(m) = I_2(m)I_4(m)\exp \left[ -\frac{1}{2} \beta_1^2 A_{3m} + \beta_1 B_{3m} \right].$$

(3.20)

Marginal posterior density of change point $m$ under non-informative prior is

$$g_2(m|X) = \frac{T_2(m)}{\sum_{m=1}^{n} T_2(m)}.$$  

(3.21)

where,

$$T_2(m)$$ is as given in (3.20).

Marginal posterior densities of $\beta_1$, say $g_1(\beta_1 | X)$ and of $\beta_2$, say $g_1(\beta_2 | X)$ are given by,

$$g_2 (\beta_1 | X) = \sum_{m=1}^{n} T_2(m) g_2(\beta_1, \beta_2, m|X)$$

(3.22)

where,

$$S_{m3}, I_2(m)$$ and $h_2^{-1}(X)$ are as given in (2.3) (3.18) and (3.15) respectively. $A_{3m}, B_{3m}$ and $k_4$ are as given in (3.16).
Remark 1: For $\mu_1 = \mu_2 = 0$, $a_1 = a_2 = 1$ the equations (3.9) to (3.11) reduce to the marginal posterior density under the non-informative prior,
\[
g_2(\beta_1, \beta_2, m) = [2\pi (n-1)]^{-1} e^{-\frac{1}{2}\beta_1^2 - \frac{1}{2}\beta_2^2}
\]
(3.13)

Remark 2: For $\mu_1 = \mu_2 = 0$, $a_1 = a_2 = 1$ the equations (3.10) to (3.12) reduce the Bayes estimates under non-informative prior (3.13).

4. BAYES ESTIMATION

4.1 Bayes Estimation Under Symmetric Loss Functions

The Bayes estimator of a generic parameter (or function thereof) $\alpha$ based on a squared error loss (SEL) function:
\[
L_1(\alpha, d) = (\alpha - d)^2,
\]
where $d$ is a decision rule to estimate $\alpha$, is the posterior mean. As a consequence, the SEL function relative to an integer parameter,
\[
L_1(m, v) = (m-v)^2, \quad m,v=0,1,2,\ldots
\]
(4.1)

Hence, the Bayesian estimate of an integer-valued parameter under the SEL function $L_1(m,v)$ is no longer the posterior mean and can be obtained by numerically minimizing the corresponding posterior loss. Generally, such a Bayesian estimate is equal to the nearest integer value to the posterior mean. So, we tell the nearest value to the posterior mean as Bayes Estimate under SEL.

The Bayes estimator of $m$ under SEL is,
\[
m^* = \frac{\sum_{m=1}^{n} m T_1(m)}{\sum_{m=1}^{n} T_1(m)}
\]
(4.2)
\[
m^{**} = \frac{\sum_{m=1}^{n} m T_2(m)}{\sum_{m=1}^{n} T_2(m)}
\]
(4.3)

Where,
\[T_1(m)\] and $T_2(m)$ are as given in (3.8) and (3.20).

Other Bayes estimators of $\alpha$ based on the loss functions
\[
L_2(\alpha, d) = |\alpha - d|
\]
\[
L_3(\alpha, d) = \begin{cases} 0, & \text{if } |\alpha - d| < \epsilon, \epsilon > 0 \\ 1, & \text{otherwise} \end{cases}
\]
is the posterior median and posterior mode, respectively.

4.2 Bayes Estimation Under Asymmetric Loss Functions

The Loss function $L_1(\alpha, d)$ provides a measure of the financial consequences arising from a wrong decision rule $d$ to estimate an unknown quantity (a generic parameter or function thereof) $\alpha$. The choice of the appropriate loss function depends on financial considerations only, and is independent from the estimation procedure used. The use of symmetric loss functions was found to be generally inappropriate, since for example, an over estimation of the reliability function is usually much more serious than an under estimation.

A useful asymmetric loss, known as the Linex loss function was introduced by Varian (1975). Under the assumption that the minimal loss occurs at $d$, the Linex loss function can be expressed as
\[
L_4(\alpha, d) = \exp \{ q_1 (d - \alpha) \} - q_1 (d - \alpha) - 1, \quad q_1 \neq 0.
\]

The sign of the shape parameter $q_1$ reflects the deviation of the asymmetry, $q_1 > 0$ if over estimation is more serious than under estimation, and vice- versa, and the magnitude of $q_1$ reflects the degree of asymmetry.

The posterior expectation of the Linex loss function is:
\[
E_{\alpha}(L_4(\alpha, d)) = \exp (q_1 d) E_{\alpha} \{ \exp (-q_1 \alpha) \} - q_1 (d - E_{\alpha} \{ \alpha \}) - 1
\]

Where $E_{\alpha}\{f(\alpha)\}$ denotes the expectation of $f(\alpha)$ with respect to the posterior density $g(\alpha|x)$. The Bayes estimate $\alpha_1^*$ is the value of $d$ that minimizes $E_{\alpha}(L_4(\alpha, d))$
\[
\alpha_1^* = -\frac{1}{q_1} \ln[E_{\alpha}(\exp(-q_1 \alpha))]
\]
provided that $E_{\alpha}\{\exp(-q_1 \alpha)\}$ exists and finite.

Minimizing expected loss function, $E_m[L_4(m, d)]$ and using posterior distribution (3.10) and (3.21), we get the Bayes estimators of $m$ using Linex loss function respectively as
\[
m_1^* = \left( -\frac{1}{q_1} \right) \ln [E e^{-q_1 m}]
\]
(4.4)
\[
m_1^{**} = \left( -\frac{1}{q_1} \right) \ln \left[ \sum_{m=1}^{n} e^{-q_1 m} \frac{T_1(m)}{\sum_{m=1}^{n} T_1(m)} \right]
\]
(4.5)

$T_1(m)$ and $T_2(m)$ are as given in (3.8) and (3.20).
Minimizing expected loss function, $E_{\beta_1} [L_4 (\beta_1, d)]$ and using posterior distribution (3.11) and (3.22), we get the Bayes estimators of $\beta_1$, using Linex loss function respectively as

$$\beta_L^* = -\frac{1}{q_1} \ln \left[ \sum_{m=1}^{\infty} e^{-q_1 \beta_1} \right]$$

(4.6) Where $A_{1m} > 0$.

Where,

$$h_1^{-1}(X) \ I_2 (m), \ k_3 \text{ and } A_{1m} \text{ are as given in (3.4), (3.6) and (3.4)a respectively.}$$

$$S_{m3} \text{ is as given in (2.3).}$$

$$\beta_L^* = -\frac{1}{q_1} \ln \left[ \sum_{m=1}^{\infty} e^{-q_1 \beta_1} \right]$$

(4.7) Where $A_{3m} > 0$.

Where,

$$h_1^{-1}(X) \ I_4 (m), \ k_4 \text{ and } A_{3m} \text{ are as given in (3.15), (3.18) and (3.16) respectively.}$$

$$S_{m3} \text{ is as given in (2.3).}$$

Minimizing expected loss function, $E_{\beta_2} [L_4 (\beta_2, d)]$ and using posterior distribution (3.12) and (3.23), we get the Bayes estimators of $\beta_2$, using Linex loss function respectively as

$$\beta_L^* = -\frac{1}{q_1} \ln \left[ \sum_{m=1}^{\infty} e^{-q_1 \beta_2} \right]$$

(4.8) Where $A_{2m} > 0$.

Where,

$$h_1^{-1}(X) \ I_1 (m), \ k_3 \text{ and } A_{2m} \text{ are as given in (3.4), (3.5) and (3.4)a respectively.}$$

$$S_{m3} \text{ is as given in (2.3).}$$

Another loss function, called general entropy (GE) loss function, proposed by Calabria and Pulcini (1994), is given by,

$$L_5 (a, d) = \frac{d}{\alpha} q_3 - q_3 \ln \left( \frac{d}{\alpha} \right) - 1$$

The Bayes estimate $\alpha_E^*$ is the value of $d$ that minimizes $E_a [L_5 (\alpha, d)]$ as

$$\alpha_E^* = [E_a (\alpha^{-q_3})]^{-1/q_3}$$

provided that $E_a (\alpha^{-q_3})$ exists and is finite.

Combining the General Entropy Loss with the posterior densities $g_i (m|x)$, i=1,2. (3.9) and (3.20), we get the estimate $m$ by means of the nearest integer value to (4.10), say $m^*_E$, as under. We get the Bayes estimates $m^*_E$ of $m$ using General Entropy loss function as

$$m^*_E = \left[ E_{\alpha} [m^{-q_3}] \right]^{-1/q_3} = \left[ \sum_{m=1}^{\infty} m^{-q_3} T_1 (m) \right]^{-1/q_3},$$

(4.10)

$$m^*_E^* = \left[ E_{\alpha} [m^{-q_3}] \right]^{1/q_3} = \left[ \sum_{m=1}^{\infty} m^{-q_3} T_2 (m) \right]^{1/q_3}.$$

Where,

$$T_1 (m) \text{ and } T_2 (m) \text{ are as given in (3.8) and (3.20).}$$

Note-1: The confluent hypergeometric function of the first kind $\text{iF}_1(a,b;x)$ (Kummer 1836), is a degenerate form of the hypergeometric function $\text{F}_1(a,b,c;x)$ which arises as a solution the confluent hypergeometric differential equation. It is also known as Kummer’s function of the first kind and denoted by $\text{iF}_1$, defined as follows.

$$\text{iF}_1(a,b;x) = \sum_{m=0}^{\infty} \frac{(a,m)x^m}{(b,m)m!} \text{ for } |x| < 1$$
With Pochhammer coefficients \((a,m) = Γ(a+m)/Γ(a)\) for \(m ≥ 1\) and \((a,0)=1\). (Arfken 1985, p. 755), also has an integral representation
\[
\text{i} F_1(a,b;x) = \int_0^1 e^{ux} u^{-a-1}(1-u)^{b-a-1} \frac{du}{B(a,b-a)}
\]
The symbols \(Γ\) and \(B\) denoting the usual functions Gamma and Beta, respectively.

When \(a\) and \(b\) are both integer, some special results are obtained. If \(a<0\), and either \(b>0\) or \(b<0\), the series yields a polynomial with a finite number of terms. If integer \(b≤0\), the function is undefined.

**Note-2:** \(p_F q\) \([(a_1), ..., (a_p)], \{(b_1),..., (b_q)\}, z\] is called a generalized hypergeometric series and defined as [Gradshteyn and Ryzhik (1965, page 1045)]
\[
p_F q\ [(a_1), ..., (a_p)], \{(b_1),..., (b_q)\}, z\] \[= \sum_{m=0}^{∞} \frac{(a_1)_{m...}(a_p)_{m}}{(b_1)_{m...}(b_q)_{m}} z^m / m!
\]

In many special cases hypergeometric \(p_F q\) is automatically converted to other functions.

For \(p=q+1\), hypergeometric \(p_F q\ \{a_1, b_1, z\}\] has a branch cut discontinuity in the complex \(z\) plane running from \(1\) to \(∞\).

Hypergeometric \(p_F q\) (Regularized) is finite for all finite values of its argument so long as \(p≤q\).

Minimizing expected loss function \(E_{\beta_1} [L_2(\beta_1, \delta)]\) and using posterior distributions (3.11) and (3.22), we get the Bayes estimates of \(\beta_1\) using General Entropy loss function respectively as,
\[
\hat{\beta}^*_E = \left\{E \left[ \beta_1^{-(q+1)} \right] \right\}^{1/q+1},
\]
\[
E \left[ \beta_1^{-(q+1)} \right] = \int_{-\infty}^{∞} \beta_1^{-(q+1)} \exp \left[ -\frac{1}{2} \beta_1^2 A_{1m} + \beta_1 B_{1m} \right] d\beta_1
\]
\[
= \frac{1}{A_{1m}} \left(1 - \frac{q_3}{2}\right) \frac{Γ\left(1 - \frac{q_3}{2}\right)}{2} \text{Hypergeometric } 1^F 1\left(\frac{q_3}{2}, 1; 2, -\frac{B_{1m}^2}{2A_{1m}}\right)
\]
\[
+ G_{2m} \Gamma \left(1 - \frac{q_3}{2}\right) \text{ Hypergeometric } 1^F 1\left(\frac{q_3}{2}, 1; 2, -\frac{B_{1m}^2}{2A_{1m}}\right)
\]
\[
G_{1m} = (-1)^{-q_2} 2^{3(-1-q_3)} \left[ -A_{1m}^2 B_{1m}^{-2q_3} B_{1m}^{-q_3} \exp \left[ \frac{B_{1m}^2}{2A_{1m}} \right] \left( A_{1m} \right)^{q_3} \left( -1 \right) \right]
\]
\[
G_{2m} = \sqrt{2} B_{1m} \left[ (-1)^{q_3} \left( A_{1m} \right)^{q_3} \left( -1 \right) \right]
\]
\[
G_{3m} = 2^{1+q_3} \left[ \left( -A_{1m}^2 B_{1m}^{-2q_3} B_{1m}^{-q_3} \exp \left[ \frac{B_{1m}^2}{2A_{1m}} \right] \left( A_{1m} \right)^{q_3} \right) \right]
\]

Where,

Hypergeometric \(1^F 1\) \(\frac{1+q_3}{2}, 1; 2, -\frac{B_{1m}^2}{2A_{1m}}\) and Hypergeometric \(p_F q\ \left\{\frac{1}{2}, 1\right\}, \left\{1 - \frac{q_3}{2}, 2, -\frac{q_3}{2}\right\}, -\frac{B_{1m}^2}{2A_{1m}}\) are

Hypergeometric functions same as explained in Note-1 and Note-2 respectively. \(S_{m3}\) is as given in (2.3). \(h_1^{-1}(X)\), \(J_2(m), B_{1m}, A_{1m}\) and \(k_3\) are as given in (3.4), (3.6) and (3.4a) and (3.9) respectively.
\[ \beta_{1E}^* = \left\{ E \left( \beta_1^{-q_3} \mid \mathcal{X} \right) \right\}^{-1/q_3}, \]

\[ E \left( \beta_1^{-q_3} \mid \mathcal{X} \right) = \int_{-\infty}^{\infty} \beta_1^{-q_3} g_1(\beta_1 \mid \mathcal{X}) \, d\beta_1 \]

\[ \begin{align*}
&= \left\{ \sum_{m=1}^{n-1} \beta_1^{-q_3} k_4 \int_{-\infty}^{\infty} e^{-q_1 \beta_1} \exp \left[-\frac{1}{2} \beta_1^2 A_{3m} + \beta_1 B_{3m} \right] d\beta_1, \right. \\
&\left. \frac{1}{2} \left\{ S_m (\sigma_1^2 - \sigma_2^2) \right\} \sigma_1^{-m} \sigma_2^{-(m-n)} h_2^{-1}(\mathcal{X}) \right\} \\
\end{align*} \]

Hence,

\[ \beta_{1E}^* = \left\{ k_4 \sum_{m=1}^{n-1} J_{3m} l_4(m) \exp \left[-\frac{1}{2} \left\{ S_m (\sigma_1^2 - \sigma_2^2) \right\} \sigma_1^{-m} \sigma_2^{-(m-n)} \right] h_2^{-1}(\mathcal{X}) \right\}^{1/q_3} \quad (4.12) \]

where,

\[ \begin{align*}
J_{3m} &= \int_{-\infty}^{\infty} \beta_1^{-q_3} \exp \left[-\frac{1}{2} \beta_1^2 A_{3m} + \beta_1 B_{3m} \right] d\beta_1 \\
&= \frac{1}{A_{3m} \left( -1 + q_3 \right)} \Gamma \left( \frac{1 - q_3}{2} \right) \text{Hypergeometric } 1F1 \left( \frac{q_3}{2}, \frac{3}{2}, -\frac{B_{3m}^2}{2A_{3m}} \right) \\
&+ \frac{k_{2m} \Gamma \left( \frac{1 - q_3}{2} \right) \text{Hypergeometric } 1F1 \left( \frac{1 + q_3}{2}, \frac{3}{2}, -\frac{B_{3m}^2}{2A_{3m}} \right)}{A_{3m} \left( -1 + q_3 \right)} \\
&+ \frac{A_{3m} \left( -1 + q_3 \right) \exp \left[- \frac{A_{3m}^2 B_{3m}^2}{2A_{3m}} \right]}{A_{3m} \left( -1 + q_3 \right)} \left\{ A_{3m}^2 + 1 \right\} \end{align*} \]

\[ \begin{align*}
k_{1m} &= (-1)^{-q_3} 2^{\frac{1}{2} - q_3} \left\{ \frac{A_{3m}^2}{B_{3m}^3} \right\}^{-q_3} \left( -\frac{B_{3m}}{A_{3m}} \right)^{-q_3} \exp \left[- \frac{A_{3m}^2 B_{3m}^2}{2A_{3m}} \right] \left\{ A_{3m}^2 + 1 \right\} \\
&+ q_3 \left\{ A_{3m} \left( -1 + q_3 \right) \left( -\frac{B_{3m}}{A_{3m}} \right)^{-q_3} \left( -\frac{B_{3m}}{A_{3m}} \right)^{-q_3} \right\} \end{align*} \]

\[ \begin{align*}
k_{2m} &= \sqrt{2} B_{3m} \left\{ (-1)^{q_3} \left( -\frac{A_{3m}}{B_{3m}} \right)^{q_3} \left( -\frac{B_{3m}}{A_{3m}} \right) \left( -\frac{B_{3m}}{A_{3m}} \right) + 1 \right\} \\
k_{3m} &= 2^{\frac{1}{1 + q_3}} \left\{ \left( -\frac{A_{3m}}{B_{3m}} \right)^{q_3} \left( -\frac{B_{3m}}{A_{3m}} \right)^{q_3} \left( -\frac{B_{3m}}{A_{3m}} \right)^{q_3} \right\} \\
\end{align*} \]

Where,

\[ \begin{align*}
&\text{Hypergeometric } 1F1 \left( \frac{1 + q_3}{2}, \frac{3}{2}, -\frac{B_{3m}^2}{2A_{3m}} \right) \text{ and Hypergeometric } 1F1 \left( \frac{1}{2}, 1, -\frac{B_{3m}^2}{2A_{3m}} \right) \text{ are Hypergeometric functions same as explained in Note-1 and Note-2 respectively.} \\
&\text{h}_1(\mathcal{X}) \text{, } l_4(m) \text{, } B_{3m} \text{, } A_{3m} \text{ and } k_{4m} \text{ are as given in (3.15), (3.18) and (3.16) respectively.} \\
&\text{S}_m \text{ is as given in (2.3).} \\
\end{align*} \]

Minimizing expected loss function \( E[L_3(\beta_2, d)] \) and using posterior distributions (3.12) and (3.23), we get the Bayes estimates of \( \beta_2 \) using General Entropy loss function respectively as,

\[ \beta_{2E}^* = \left\{ E \left( \beta_2^{-q_3} \mid \mathcal{X} \right) \right\}^{-1/q_3} \]

\[ E \left( \beta_2^{-q_3} \mid \mathcal{X} \right) = \int_{-\infty}^{\infty} \beta_2^{-q_3} g_2(\beta_2 \mid \mathcal{X}) \, d\beta_2 \]

\[ \begin{align*}
&= \sum_{m=1}^{n-1} \beta_2^{-q_3} \exp \left[-\frac{1}{2} \beta_2^2 A_{2m} + \beta_2 B_{2m} \right] \\
&\beta_2 B_{2m} \right] l_1(m) \exp \left[-\frac{1}{2} \left\{ S_m (\sigma_1^2 - \sigma_2^2) \right\} \sigma_1^{-m} \sigma_2^{-(m-n)} \right] h_1^{-1}(\mathcal{X}) \end{align*} \]

Hence,

\[ \beta_{2E}^* = \sum_{m=1}^{n-1} \int_{-\infty}^{\infty} \beta_2^{-q_3} \exp \left[-\frac{1}{2} \left\{ S_m (\sigma_1^2 - \sigma_2^2) \right\} \sigma_1^{-m} \sigma_2^{-(m-n)} \right] h_1^{-1}(\mathcal{X}) \right\}^{1/q_3} \quad (4.13) \]

where,

\[ \begin{align*}
J_{2m} &= \sum_{m=1}^{n-1} \beta_2^{-q_3} \exp \left[-\frac{1}{2} \beta_2^2 A_{2m} + \beta_2 B_{2m} \right] d\beta_2 \\
&= \frac{1}{A_{2m} \left( -1 + q_3 \right)} \Gamma \left( \frac{1 - q_3}{2} \right) \text{Hypergeometric } 1F1 \left( \frac{q_3}{2}, \frac{1}{2}, -\frac{B_{2m}^2}{2A_{2m}} \right) \end{align*} \]
\[ G_{5m} = \sqrt{2B_{2m}} \left[ (-1)^{q_3} \left( \frac{A_{2m}^2}{B_{2m}^2} \right)^{q_3} \left( -\frac{B_{2m}}{A_{2m}} \right)^{q_3} - 1 \right] \]

\[ G_{6m} = 2^{1+q_3} \left[ \left( -\frac{A_{2m}^2}{B_{2m}^2} \right)^{q_3} B_{2m} \left( -\frac{B_{2m}}{A_{2m}} \right)^{q_3} \left( \frac{B_{2m}}{A_{2m}} \right)^{q_3} \right] \]

Hypergeometric 1F1 \( \left( \frac{1+q_3}{2}, \frac{3}{2}, -\frac{B_{2m}^2}{A_{2m}^2} \right) \) and Hypergeometric \( p^r q \left( \left\{ \frac{1}{2}, 1 \right\}, \left\{ 1 - \frac{q_3}{2}, q_3 \right\}, -\frac{B_{2m}^2}{A_{2m}^2} \right) \) are Hypergeometric functions same as explained in Note 1 and Note 2 respectively. \( h_{1,2}^{-1}(X) \), \( l_1(m) \), \( A_{2m}, B_{2m} \) and \( k_5 \) are as given in (3.4), (3.5) and (3.4)a respectively. \( S_{m3} \) is as given in (2.3).

\[ E \left( \beta_2^{-q_3} \right) = \int_{0}^{\infty} \beta_2^{-q_3} g_2(\beta_2; X) \, d\beta_2 \]

\[ = k_4 \sum_{m=1}^{n-1} \int_{0}^{\infty} \beta_2^{-q_3} \exp \left[ -\frac{1}{2} \beta_2^2 A_{4m} + \beta_2 B_{4m} \right] l_3(m) \exp \left[ -\frac{1}{2} \left( S_{m3} \right) \sigma_1^{-2} \sigma_2^{-2} \right] \sigma_1^{-m} \sigma_2^{-(n-m)} \cdot h_{2,1}^{-1}(X) \]

Hence,

\[ \beta_2^{*E} = \left( k_4 \sum_{m=1}^{n-1} J_{2m} \right) l_3(m) \exp \left[ -\frac{1}{2} \left( S_{m3} \right) \sigma_1^{-2} \sigma_2^{-2} \right] \sigma_1^{-m} \sigma_2^{-(n-m)} \cdot h_{2,1}^{-1}(X) \]

where,

\[ J_{4m} = \int_{-\infty}^{\infty} \sum_{m=1}^{n-1} \beta_2^{-q_3} \exp \left[ -\frac{1}{2} \beta_2^2 A_{4m} + \beta_2 B_{4m} \right] \, d\beta_2 \]

\[ = \frac{1}{A_{4m}(-1+q_3)} k_{4m} \Gamma \left( \frac{1-q_3}{2} \right) \left( \frac{1}{2}, \frac{3}{2}, -\frac{B_{4m}^2}{2A_{4m}} \right) \]

\[ + k_{5m} \Gamma \left( \frac{1-q_3}{2} \right) \left( \frac{1}{2}, \frac{3}{2}, -\frac{B_{4m}^2}{2A_{4m}} \right) \]

\[ k_{4m} = (-1)^{-q_3} 2^{\frac{1}{2}(1-q_3)} \left( \frac{A_{4m}^2}{B_{4m}^2} \right)^{q_3} \left( -\frac{B_{4m}}{A_{4m}} \right)^{q_3} \exp \left( \frac{B_{4m}^2}{2A_{4m}} \right) \left( A_{4m} \right)^{q_3} (-1) \]

\[ + q_3 \left\{ \sqrt{A_{4m}} \left( -A_{4m} \right)^{q_3} \left( -\frac{B_{4m}}{A_{4m}} \right)^{q_3} + 1 \right\} \]

\[ k_{5m} = \sqrt{2B_{4m}} \left[ (-1)^{q_3} \left( \frac{A_{4m}^2}{B_{4m}^2} \right)^{q_3} \left( -\frac{B_{4m}}{A_{4m}} \right)^{q_3} - 1 \right] \]

\[ k_{6m} = 2^{1+q_3} \left[ \left( \frac{A_{4m}^2}{B_{4m}^2} \right)^{q_3} B_{4m} \left( -\frac{B_{4m}}{A_{4m}} \right)^{q_3} + \left( \frac{B_{4m}^2}{A_{4m}} \right)^{q_3} \right] \]

Where,

Hypergeometric 1F1 \( \left( \frac{1+q_3}{2}, \frac{3}{2}, -\frac{B_{4m}^2}{2A_{4m}} \right) \) and Hypergeometric \( p^r q \left( \left\{ \frac{1}{2}, 1 \right\}, \left\{ 1 - \frac{q_3}{2}, \frac{q_3}{2} \right\}, -\frac{B_{4m}^2}{2A_{4m}} \right) \) are Hypergeometric functions same as explained in Note 1 and
Note-2 respectively. $h_2^k(X)$, $J_3(m)$, $A_{4m}$, $B_{4m}$ and $k_4$ are as given in (3.15), (3.17) and (3.16) respectively. $S_{m3}$ is as given in (2.3).

5. ILLUSTRATION

Let us consider AR(1) model as

$$X_1 = \begin{cases} 
0.1X_{t-1} + \epsilon_i, & i = 1, 2, \ldots, 10 \\
0.3X_{t-1} + \epsilon_i, & i = 11, 12, \ldots, 20 
\end{cases}$$

Where, $\epsilon_i$’s are independently distributed distributions. We have generated 20 random observations from proposed AR(1) model given in (2.1). The first ten observations are from normal distribution with $\sigma_1 = 1$ and next 10 are from normal distribution with $\sigma_2 = 4$. $\beta_1$ and $\beta_2$ themselves were random observations from normal distributions with prior means $\mu_1 = 0.1$, $\mu_2 = 0.3$ and variances $a_1 = 0.1$ and $a_2 = 0.1$. These observations are given in table 1.

<table>
<thead>
<tr>
<th>Table 1: Generated observations from proposed AR(1) model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
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<tr>
<td>7</td>
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<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Posterior distributions of m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3: Cumulative Posterior distributions of m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

The values of posterior density and cumulative posterior probability of $m$ are calculated. The results are shown in Table 2 and 3. We have calculated posterior mean of $m$, $\beta_1$, and $\beta_2$ and the posterior median of $m$. posterior mode appears to be a bad estimator of $m$. For a comparative purpose point of view, estimators under the non-informative prior are also calculated. The results are shown in table 4.

<table>
<thead>
<tr>
<th>Table 4: The values of Bayes estimates of change point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior Density</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Inforative</td>
</tr>
<tr>
<td>Non informative</td>
</tr>
</tbody>
</table>
We compute the Bayes estimates \( m^*_1, m^*_E \) of \( m_1, m_2 \) respectively for the data given in Table 5. For a comparative purpose, Bayes estimates under the non-informative prior and asymmetric loss functions are also computed and results are shown in Table 5 and shows that \( m^*_1, m^*_E \) are robust with respect to the change in the shape parameter of GE loss function.

### Table 5: The Bayes estimates using Asymmetric loss functions.

<table>
<thead>
<tr>
<th>Prior Density</th>
<th>Shape parameter</th>
<th>Bayes estimates of change point</th>
<th>Bayes estimates under General Entropy Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q_1 ) ( q_1 )</td>
<td>( m^*_1 )</td>
<td>( m^*_E )</td>
</tr>
<tr>
<td>Informative</td>
<td>0.09 0.09</td>
<td>10 9</td>
<td>0.33 0.32</td>
</tr>
<tr>
<td>0.10 0.10</td>
<td>10 9</td>
<td>0.32 0.31</td>
<td>0.13 0.11</td>
</tr>
<tr>
<td>0.20 0.20</td>
<td>9 9</td>
<td>0.31 0.30</td>
<td>0.11 0.10</td>
</tr>
</tbody>
</table>

### Table 6: Posterior mean \( m^* \) for the data given in Table 7.1.

<table>
<thead>
<tr>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( m^* )</th>
<th>( m^*_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.6</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.1</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.07</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.1</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.3</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

The results are shown in Table 6, which lead to conclusion that \( m^* \) and \( m^*_E \) are robust with respect to the correct choice of the prior density of \( \beta_1 (\beta_2) \) and a wrong choice of the prior density of \( \beta_2 (\beta_1) \). Moreover, they are also robust with respect to the change in the shape parameter of GE loss function.

### 6. Sensitivity of Bayes Estimates

In this section, we study the sensitivity of the Bayes estimates, obtained in section 7.4 and section 7.5 with respect to change in the prior of the parameter. The means \( \mu_1, \mu_2 \) and variances \( \sigma_1, \sigma_2 \) of normal prior have been used as prior information. Following Calabria and Pulcini (1996), we also assume the prior information to be correct if the true value of \( \beta_1 (\beta_2) \) is close to prior mean \( \mu_1 (\mu_2) \) and is assumed to be wrong if \( \beta_1 (\beta_2) \) is far from \( \mu_1 (\mu_2) \). We have computed \( m^* \) and \( m^*_E \) and given in section 4 for the data given in Table 1 with common value of \( q_1 = 0.1 \) and \( q_2 = 0.1 \) respectively, for \( q_3 = 0.9 \), considering different values of \( (\mu_1, \mu_2) \) and result are shown in Table 6.

### Table 7: The Bayes estimates under the correct prior consideration.

The value of shape parameter of the general entropy loss and Linex loss used in simulation study for change point is taken as 0.1. We have also simulated several samples from AR 1 model explained section 2 with \( m=10, n=20, \beta_1=0.1, \mu_2=0.3 \) and \( \beta_2=0.24, 0.3, 0.29 \) for each \( \beta_1 \) and \( \beta_2 \). 1000 pseudo random samples with \( m=10 \) and \( n=20 \) have been simulated and Bayes estimators of change point \( m \) and autoregressive coefficients \( \beta_1 \) and \( \beta_2 \) using \( q_1 = 0.9 \) has been computed for different prior means \( \mu_1 \) and \( \mu_2 \). We also obtained the frequency distributions of Bayes estimates of autoregressive coefficients given in section 4 with the correct prior consideration for generated 10,000 different random samples. The results are shown in Table 8 and 9.
8. CONCLUSIONS

Our numerical study showed that the Bayes estimators posterior mean of m, and \( m^*_E \) are robust with respect to the correct choice of the prior specifications on \( \beta_1 \) and \( \beta_2 \), and wrong choice of the prior specifications on \( \beta_2 \). Moreover, posterior mean of m is sensitive when prior specifications on both \( \beta_1 \) and \( \beta_2 \) deviate simultaneously from the true values. Numerical study also showed that posterior mean of m is sensitive when prior specifications on both \( \beta_1 \) and \( \beta_2 \) deviate simultaneously from the true values.

9. REFERENCES