BAYESIAN INFERENCE ON MIXTURE OF GEOMETRIC WITH DEGENERATE DISTRIBUTION: ZERO INFLATED GEOMETRIC DISTRIBUTION

Mayuri Pandya, Hardik Pandya & Smita Pandya
Department of Statistics, Bhavnagar University
University Campus, Near Gymkhana
Bhavnagar-364002, India

ABSTRACT

Power series distributions form a useful subclass of one-parameter discrete exponential families suitable for modeling count data. A zero-inflated Geometric distribution is a mixture of a Geometric distribution and a degenerate distribution at zero, with a mixing probability \( p \) for the degenerate distribution. This distribution is useful for modeling count data that may have extra zeros. A sequence of independent count data \( X_1, X_2, X_m, X_{m+1}, \ldots \), \( X_n \) were observed from a zero-inflated Geometric distribution with probability mass function \( f^*(x \mid p_1, \theta_1) \), but later it was found that there was a change in the system at some point \( m \) and it is reflected in the sequence after \( X_m \) by change in probability mass function \( f^*(x \mid p_2, \theta_2) \). The Bayes estimators of \( m, \theta_1, p_1, \theta_2, p_2 \) are derived under different asymmetric loss functions. The effects of correct and wrong prior information on the Bayes estimates are studied.

Keywords: Bayesian Inference, Estimation, Zero-inflated Geometric, Degenerate, Mixture of Distribution, Change Point.

1. INTRODUCTION

Power series distributions form a useful subclass of one-parameter discrete exponential families suitable for modelling count data. A zero-inflated power series distribution is a mixture of a power series distribution and a degenerate distribution at zero, with a mixing probability \( p \) for the degenerate distribution. This distribution is useful for modelling count data that may have extra zeros. A. Bhattacharya ET. al. (2008) has presented a Bayesian test for the problem based on recognizing that the parameter space can be expanded to allow \( p \) to be negative. They also find that using a posterior probability as a test statistic has slightly higher power on the most important ranges of the sample size \( n \) and parameter values than the score test and likelihood ratio test in simulations.

Models for count data often fail to fit in practice because of the presence of more zeros in the data than is explained by a standard model. This situation is often called zero inflation because the number of zeros is inflated from the baseline number of zeros that would be expected in, say, a one-parameter discrete exponential family. Zero inflation is a special case of over dispersion that contradicts the relationship between the mean and variance in a one-parameter exponential family. One way to address this is to use a two-parameter distribution so that the extra parameter permits a larger variance.

Johnson, Kotz and Kemp (1992) discuss a simple modification of a power series (PS) distribution \( f^*(\theta) \) to handle extra zeros. An extra proportion of zeros, \( p \), is added to the proportion of zeros from the original discrete distribution, while decreasing the remaining proportions in an appropriate way. So the zero inflated PS distribution is defined as

\[
f^*(y \mid p, \theta) = \begin{cases} 
    p + (1 - p)f(0 \mid \theta), & y = 0 \\
    (1 - p)f(y \mid \theta), & y > 0 
\end{cases}
\]

Where \( \theta \in \Theta \), the parameter space and the mixing parameter \( p \) range over the interval,

\[
-\frac{f(0 \mid \theta)}{(1 - f(0 \mid \theta))} < p < 1
\]

This allows the distribution to be well defined for certain negative values of \( p \), depending on \( \theta \). Although the mixing interpretation is lost when \( p < 0 \), these values have a natural interpretation in terms of zero-deflation, relative to a PS model. Correspondingly, \( p > 0 \) can be regarded as zero inflation relative to a PS model. Note that the PS family contains all discrete one-parameter exponential families so an appropriate choice of PS model in (1) permits any desired interpretation for the data corresponding to the second term. The first term allows an extra proportion \( p \) of
zeros to be added to the discrete PS distribution zero inflated Geometric distribution with parameter \((p, \theta)\) is given by:

$$f^*(x|p, \theta) = \begin{cases} 
(p + (1 - p)(1 - \theta), & x = 0 \\
(1 - p)(1 - \theta)^x, & x \geq 1, \ldots 
\end{cases}$$  \hspace{1cm} (2)

A statistical model is specified to represent the distribution of changing count data that may have extra zeros and statistical inferences are made on the basis of this model. Counting data are often subject to random fluctuations. It may happen that at some point of time instability in the sequence of count data is observe and number of extra zeroes are changed. The problem of study in when and where this change has started occurring is called change point inference problem. Bayesian ideas may play as important role in the study of such change point problem and has been often proposed as veiled alternative to classical estimation procedure. The monograph of Broemeling and Tsurumi [1987] on structural change, Jani and Pandya [1999] Ebahimi and Ghosh [2001] and a survey by Jack [1983] Pandya and Jani [2006] Pandya and Bhatt [2007] Mayuri Pandya and Prbha Jadav( [2008] [2010]) are useful reference.

In this paper we have proposed change point model on Zero Inflated Geometric distribution to represent the distribution of count data with change point \(m\) and have obtain Bays estimates of \(m, p_1, \text{and } p_2\).

2. PROPOSED CHANGE POINT MODEL ON ZERO INFLATED GEOMETRICAL DISTRIBUTION

Let \(X_1, X_2, \ldots, X_n\) \((n \geq 3)\) be a sequence of observed count data. Let first \(m\) observations \(X_1, X_2, \ldots, X_m\) come from the Zero Inflated Geometric distribution with probability mass function \(f^*(x_1|p_1, \theta_1)\), and later \((n-m)\) observations \(X_{m+1}, X_{m+2}, \ldots, X_n\) coming from the Zero Inflated Geometric distribution with probability mass function \(f^*(x_i|p_2, \theta_2)\),

$$f^*(x_i|p_2, \theta_2) = \begin{cases} 
(p_2 + (1 - p_2)(1 - \theta_2), & x_i = 0 \\
(1 - p_2)(1 - \theta_2)^{x_i}, & x_i \geq 1, \ldots 
\end{cases}$$  \hspace{1cm} (3)

Where, \(I(x_i) = \begin{cases} 1, & x_i = 0 \\
0, & x_i > 0 \end{cases}\)

And \(m\) is change point. \(p_1\) and \(p_2\) are proportions.

3a. POSTERIOR DISTRIBUTION FUNCTIONS USING INFORMATIVE PRIOR

The ML method, as well as other classical approaches is based only on the empirical information provided by the data. However, when there is some technical knowledge on the parameters of the distribution available, a Bayes procedure seems to an attractive inferential method. The Bayes procedure is based on a posterior density, say \(g(\theta_1, \theta_2, p_1, p_2, m|X)\), which is proportional to the product of the likelihood function \(L(\theta_1, \theta_2, p_1, p_2, m|X)\) with a prior joint density, say \(g(\theta_1, \theta_2, p_1, p_2, m)\) representing the uncertain on the parameters values.

The likelihood function of the parameters \(\theta_1, \theta_2, p_1, p_2, m\) given the sample information \(X = \{X_1, X_2, \ldots, X_m, X_{m+1}, \ldots, X_n\}\) is given by,

$$L(\theta_1, \theta_2, p_1, p_2, m|X) = (p_1 + (1 - p_1)(1 - \theta_1))^{dm} (1 - \theta_1)^{m-dm} (1 - p_1)^{m-dm} \theta_1^{Sm}$$

$$\{p_2 + (1 - p_2)(1 - \theta_2)^{dn-dm} (1 - \theta_2)^{n-m-dn+dm} (1 - p_2)^{n-m-dn+dm} \theta_2^{Sn-Sm}\}$$  \hspace{1cm} (4)

Where,

$$\Sigma_{i=1}^m I(x_i) = dm, \quad I(x_i) = \begin{cases} 1, & x_i = 0 \\
0, & x_i > 0 \end{cases}$$

$$\Sigma_{i=1}^m (1 - I(x_i)) = m - dm,$$

$$\Sigma_{i=m+1}^n (1 - I(x_i)) = n - m - dm,$$

$$\Sigma_{i=1}^m x_i (1 - I(x_i)) = Sm,$$

$$\Sigma_{i=m+1}^n x_i (1 - I(x_i)) = Sn - Sm.$$  \hspace{1cm} (5)
As in Broemeling et. al. (1987), we suppose the marginal prior distribution of m to be discrete uniform over the set \{1, 2, 3, ..., n-1\).

\[ g_1(m) = \frac{1}{n-1} \]

Let the marginal prior distribution of \( \theta_1 \) and \( \theta_2 \) to be beta Distribution with mean \( \mu_i \) \( i = 1, 2 \) and common standard deviation \( \sigma_1 \).

\[ g_1(\theta_i) = \frac{(1-\theta)^{a_i-1}\theta^{b_i-1}}{\beta(a_i,b_i)} \quad a_i, b_i > 0 \quad i = 1, 2 \]

If the prior information is given in terms of the prior means \( \mu_1, \mu_2 \) and a common standard deviation \( \sigma_1 \) then the beta parameters \( a_i, b_i, i=1, 2 \) can be obtained by solving (6).

\[ a_i = \sigma_1^{-1} [(1-\mu_i)\mu_i^2 - \mu_i\sigma_1] \]

\[ b_i = \mu_i^{-1}(1-\mu_i)a_i \quad i = 1, 2 \]

Let the marginal prior distribution of \( p_1 \) and \( p_2 \) to be beta Distribution with prior mean \( \mu_i \) \( i = 3, 4 \) and common standarddeviation \( \sigma_2 \). So,

\[ g_1(p_j) = \frac{(1-p_j)^{c_j-1}p_j^{d_j-1}}{\beta(c_j,d_j)} \quad c_j, d_j > 0 \quad j = 1, 2 \]

If the prior information is given in terms of the prior means \( \mu_3, \mu_4 \) and a common standard deviation \( \sigma_2 \) then the beta parameters \( c_i, d_i, i=1, 2 \) can be obtained by solving (6).

\[ c_j = \sigma_2^{-1} [(1-\mu_i)\mu_i^2 - \mu_i\sigma_2] \quad i = 3, 4 \quad j = 1, 2 \]

\[ d_j = \mu_i^{-1}(1-\mu_i)c_j \quad i = 1, 2 \]

We assume that \( \theta_1, \theta_2, p_2, p_1 \) and \( m \) are priori independent. The joint prior density is say, \( g_1(\theta_1, \theta_2, p_2, p_1, m) \) obtained as,

\[ g_1(\theta_1, \theta_2, p_2, p_1, m) = k(1-\theta_1)^{a_1-1}\theta_1^{b_1-1}(1-\theta_2)^{a_2-1}\theta_2^{b_2-1}(1-p_1)^{d_1-1}p_1^{c_1-1}(1-p_2)^{d_2-1}p_2^{c_2-1}(1-p_2^{d_2-1})p_2^{d_2-1} \]

Where,

\[ k = \frac{1}{(n-1)^n a_1 b_1 a_2 b_2 c_1 d_1 c_2 d_2) \]

Note 1. The Gauss hyper geometric function in three parameter a, b and c, denoted by \( _2F_1 \), is defined by,

\[ _2F_1(a, b, c; x) = \sum_{m=0}^{\infty} \frac{(a, m)(b, m)x^m}{(c, m)m!} \quad \text{for } |x| < 1 \]

With Pochhammer coefficients,

\[ (a, m) = \frac{\Gamma(a + m)}{\Gamma(a)} \quad \text{for } m \geq 1 \text{ and } (a, 0) = 1 \]

This function has the following integral representation,

\[ _2F_1(a, b, c; x) = \int_{0}^{1} \frac{[u^{a-1}(1-u)^{c-a-1}(1-xu)^{-b}]}{B(a, c-a)} \ du \]

The symbols \( \Gamma \) and \( B(\cdot) \) denoting the usual functions Gamma and Beta respectively. This function is a solution to a hyper geometric differential equation. It is known as Gauss series or the Kummer series.

The Joint posterior density of parameters \( \theta_1, \theta_2, p_2, p_1 \) and \( m \) is obtained using the likelihood function (4) and the prior joint density of the parameters (8),

\[ g_1(\theta_1, \theta_2, p_2, p_1, m | X) \]

\[ \propto (p_1 + (1-p_1)(1-\theta_1))^m (1-p_1)^m-dm+d_1-1p_1 \quad \theta_1^{m-b_1-1} (p_2 + (1-p_2)(1-\theta_2))^m-dm 
\]

\[ (1-p_2)^n-m-dn+d_2-1p_2^{c_2-1} (1-\theta_2)^{n-m-dn+d_2-1} \theta_2^{d_2-1} / h_1(X) \]

And \( h_1(X) \) is the marginal posterior density of \( X \),

\[ h_1(X) = \sum_{m=1}^{n-1} \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} L(\theta_1, \theta_2, p_1, p_2, m | X) g(\theta_1, \theta_2, p_1, p_2, m) d\theta_1 d\theta_2 dp_1 dp_2 \]

\[ = k \sum_{m=1}^{n-1} l_1(m) l_2(m) \]

Where,

\[ l_1(m) = \Gamma c_1 \Gamma(m - dm + d_1) \int_{0}^{1} (1-\theta_1)^{m+a_1-1} \theta_1^{s_1+b_1-1} \]
\[ 2F_1[c_1, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}], h_1^{-1}(X) \]

Where \( 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \) is a hypergeometric function same as explained in note 1.

The marginal density of \( \theta_1 \) is obtained using the joint posterior density of \( \theta_1 \) and \( \theta_2 \) and integrating with respect to \( \theta_2 \). We get,

\[ g_1(\theta_1|X) = k \sum_{m=1}^{n-1} \int_0^1 g_1(\theta_1, \theta_2|X) \, d\theta_2 \]

\[ = k \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 \Gamma(m - dm + d_1) \Gamma(n - m - dn + dm + d_2) \]

\[ (1 - \theta_1)^{m+a_1-1} \theta_1^{n+b_1-1} 2F_1[c_1, -(dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}] \]

\[ \int_0^1 (1 - \theta_2)^{n-m+a_2-1} \theta_2^{n-b_2-1} n^{-m} \, d\theta_2, \]

\[ 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}, h_1^{-1}(X)] \]

The marginal density of \( \theta_2 \) is obtained as,

\[ g_1(\theta_2|X) = k \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 \Gamma(m - dm + d_1) \Gamma(n - m - dn + dm + d_2) \]

\[ (1 - \theta_2)^{n-m+a_2-1} \theta_2^{n+b_2-1} 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \]

\[ \int_0^1 (1 - \theta_1)^{m+a_1-1} \theta_1^{n+b_1-1} n^{-m} \, d\theta_1, \]

\[ 2F_1[c_1, -(dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}], h_1^{-1}(X) \]

Where \( 2F_1[c_1, -(dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}] \) and \( 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \) are hypergeometric functions.

The joint posterior density of \( p_1 \) and \( p_2 \) is obtained as,
\[ g_1(p_1, p_2 | X) = k \sum_{m=1}^{n-1} \int_0^1 \int_0^1 g_1(\theta_1, \theta_2, p_2, p_1, m | X) \, d\theta_1 \, d\theta_2 \]

\[ = k \sum_{m=1}^{n-1} \{(1 - p_1)^{n-dm + dm + 1 - p_1} p_1 c_1^{-1} \}
\[ \Gamma(m - dm + a_1) \Gamma(Sm + b_1) \Gamma(n - m - dn + dm + a_2) \Gamma(Sn - Sm + b_2) \]
\[ \sum_{m=1}^{n-1} \{(1 - p_2)^{n-m-dm + dm + z - 1} p_2 c_2^{-1} \}
\[ g_1(p_2 | X) = k \sum_{m=1}^{n-1} \{(1 - p_2)^{n-m-dm + dm + z - 1} p_2 c_2^{-1} \}
\[ \Gamma(m - dm + a_1) \Gamma(Sm + b_1) \Gamma(n - m - dn + dm + a_2) \Gamma(Sn - Sm + b_2) \]
\[ \sum_{m=1}^{n-1} \{(1 - p_2)^{n-m-dm + dm + z - 1} p_2 c_2^{-1} \}
\[ g_1(m | X) = k \frac{l_1(m) l_2(m)}{\sum_{m=1}^{n-1} l_1(m) l_2(m)} \quad (20) \]

\[ 3b. \quad \text{POSTERIOR DISTRIBUTION FUNCTION USING NON INFORMATIVE PRIOR} \]

A non-informative prior is a prior that reflects indifference to all values of the parameter, and adds no information to that contained in the empirical data. Thus, a Bayes inference based upon non-informative prior has generally a theoretical interest only, since, from an engineering view point, the Bayes approach is very attractive for it allows incorporating expert opinion or technical knowledge in the estimation procedure. However, such a Bayes inference acquires large interest in solving prediction problems when it is extremely difficult, if at all possible, to find a classical solution for the prediction problem, because classical prediction intervals are numerically equal to the Bayes ones based on the non-informative prior density. Hence, the Bayes approach based on prior ignorance can be viewed as mathematical method for obtaining classical prediction intervals.

The joint prior density of parameters using non-informative prior is say,

\[ g_2(\theta_1, \theta_2, p_2, p_1, m) = \frac{1}{(1-\theta_1)(1-\theta_2)(1-p_1)(1-p_2)} \quad (21) \]
The joint posterior density of parameters $\theta_1, \theta_2, p_2, p_1$ and $m$ under non-informative prior is obtained using the likelihood function (4) and the joint prior density of the parameters under non-informative prior (21),

$$g_2(\theta_1, \theta_2, p_2, p_1, m|X) = \frac{g_2(\theta_1, \theta_2, p_2, p_1, m)g_2(\theta_1, \theta_2, p_2, p_1, m)}{h_2(X)}$$

$$= (p_1 + (1 - p_1)(1 - \theta_1))^{dm} (1 - p_1)^{m - dm + 1} (1 - \theta_1)^{m - dm - 1} \theta_1^{Sm - 1} \theta_2^{Sn - Sm - 1}$$

Where, $h_2(X)$ is the marginal density of $X$ under non-informative priors.

$$h_2(X) = k_1 \sum_{m=1}^{n-1} I_3(m) I_4(m)$$

(24)

Where,

$$I_3(m) = \Gamma(m - dm) \int_0^1 (1 - \theta_1)^{m - 1} \theta_1^{Sm - 1} \Gamma(m - dm)$$

$$= {}_2F_1[1, -dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] d\theta_1$$

(25)

$$I_4(m) = \Gamma(n - m - dn + dm) \int_0^1 (1 - \theta_2)^{n - m - 1} \theta_2^{Sn - Sm - 1}$$

$$= {}_2F_1[1, dm - dn, 1 + dm - dn + dm - m + n, \frac{\theta_2}{\theta_2 - 1}] d\theta_2$$

(26)

Now, the joint posterior density of $\theta_1 and \theta_2$ and of $p_1 and p_2$ are obtained as

$$g_2(\theta_1, \theta_2|X) = k_1 \sum_{m=1}^{n-1} k_2 (1 - \theta_1)^{m - 1} \theta_1^{Sm - 1}$$

$$= {}_2F_1[1, -dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] (1 - \theta_2)^{n - m - 1} \theta_2^{Sn - Sm - 1}$$

(27)

$$g_2(p_1, p_2|X) = k_1 \sum_{m=1}^{n-1} (1 - p_1)^{m - dm - 1} (1 - p_2)^{n - m - dn + dm + 1}$$

$$\Gamma(m - dm) \Gamma(Sm) \Gamma(n - m - dn + dm) \Gamma(Sn - Sm)$$

$$= {}_2F_1[1, dm - dn, 1 + dm - dn - m + n, \frac{\theta_2}{\theta_2 - 1}] h_2^{-1}(X)$$

Where,

$$k_2 = \Gamma(m - dm) \Gamma(n - m - dn + dm)$$

(29)

$$= {}_2F_1[1, Sn - Sm, -dn + dm, n - m - dn + dm + Sn - Sm, 1 - p_2] h_2^{-1}(X)$$

(30)

The marginal posterior density of $\theta_1, \theta_2, p_1$ and $p_2$ are obtained as

$$g_2(\theta_1|X) = k_1 \sum_{m=1}^{n-1} k_2 (1 - \theta_1)^{m - 1} \theta_1^{Sm - 1}$$

$$= {}_2F_1[1, -dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}]$$

$$\int_0^1 (1 - \theta_2)^{n - m - 1} \theta_2^{Sn - Sm - 1} (-\theta_2)^{dn - dm}$$

$$= {}_2F_1[1, dm - dn, 1 + dm - dn - m + n, \frac{\theta_2}{\theta_2 - 1}] d\theta_2 h_2^{-1}(X)$$

(30)

$$g_2(\theta_2|X) = k_1 \sum_{m=1}^{n-1} k_2 (1 - \theta_2)^{n - m - 1} \theta_2^{Sn - Sm - 1}$$

58
\[ 2F_1[1, dm - dn, 1 + dm - dn - m + n, \frac{\theta_1}{\theta_2 - 1}] \]

\[
\int_0^1 (1 - \theta_1)^m - 1 \theta_1^{m-1} 2F_1[1, -dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] \, d\theta_1 \, h_2^{-1}(X) \quad (31) \]

\[ g_2(p_1 | X) = k_1 \sum_{n=1}^{n-1} \Gamma(m - dm) \Gamma(Sm) \Gamma(n - m - dn + dm) \Gamma(Sn - Sm) \]

\[
(1 - p_1)^{m - dm - 1} 2F_1[Sm, -dn, m - dm + Sm, 1 - p_1] \]

\[ \int_0^1 (1 - p_2)^{n - m - dn + dm - 1} 2F_1[Sn - Sm, -dn + dm, n - m - dn + dm + Sn - Sm, 1 - p_2] \, d\theta_2 \, h_2^{-1}(X) \quad (32) \]

\[ g_2(p_2 | X) = k_1 \sum_{n=1}^{n-1} \Gamma(m - dm) \Gamma(Sm) \Gamma(n - m - dn + dm) \Gamma(Sn - Sm) \]

\[
(1 - p_2)^{n - m - dn + dm - 1} 2F_1[Sm, -dn + dm, m - dm + Sm, 1 - p_1] \]

\[ \int_0^1 (1 - p_1)^{m - dm - 1} 2F_1[Sm, -dn, m - dm + Sm, 1 - p_1] \, d\theta_1 \, h_2^{-1}(X) \quad (33) \]

Where \( 2F_1[Sm, -dn, m - dm + Sm, 1 - p_1] \) are hypergeometric functions as explained in note 1.

The marginal posterior density of \( m \) is obtained using the joint posterior density of \( \theta_1, \theta_2, p_1, p_2 \) and \( m \) and integrating with respect to \( \theta_1, \theta_2, p_1, p_2 \) leads to the marginal posterior density of change point \( m \),

\[ g_2(m | X) = k_1 I_3(m) I_4(m) h_2^{-1}(X) \]

\[ = \frac{I_3(m) I_4(m)}{\sum_{m=1}^{n-1} I_3(m) I_4(m)} \quad (34) \]

4. BAYES ESTIMATES USING INFORMATIVE AND NON-INFORMATIVE PRIOR UNDER SYMMETRIC LOSS FUNCTION

The Bayes estimate of a generic parameter (or function there of) \( \alpha \) based on a squared error loss (SEL) function:

\[ L_1(\alpha, d) = (\alpha - d)^2, \]

Where, \( d \) is a decision rule to estimate \( \alpha \), is the posterior mean.

Bayes estimates of \( m, p_1, p_2, \theta_1 \) and \( \theta_2 \) under SEL and informative prior and non-informative prior are

\[ m^* = \frac{\sum_{m=1}^{n-1} m I_1(m) I_2(m)}{\sum_{m=1}^{n-1} I_1(m) I_2(m)} \quad (35) \]

\[ m^{**} = \frac{\sum_{m=1}^{n-1} m^2 I_2(m) I_4(m)}{\sum_{m=1}^{n-1} I_2(m) I_4(m)} \quad (36) \]

\[ \theta_1^* = [k \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 \Gamma (m - dm + d_1) \Gamma (n - m - dn + dm + d_2) \]

\[ \int_0^1 (1 - \theta_1)^{m+a_1-1} \theta_1^{a_1} 2F_1[c_1, -dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}] \, d\theta_1 \]

\[ \int_0^1 (1 - \theta_2)^{n-m+a_2-1} \theta_2^{a_2} 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \, d\theta_2 \]

\[ 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \, h_1^{-1}(X) \quad (37) \]

\[ \theta_2^* = [k \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 \Gamma (m - dm + d_1) \Gamma (n - m - dn + dm + d_2) \]

\[ \int_0^1 (1 - \theta_2)^{n-m+a_2-1} \theta_2^{a_2} 2F_1[c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \, d\theta_2 \]

\[ 2F_1[c_1, -dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}] \, h_1^{-1}(X) \quad (38) \]
\[ p_1^n = \left[ k \sum_{m=1}^{n-1} \Gamma(m - dm + a_1)I(m - dm + a_2) \Gamma(n - dm + a_1 + a_2) \right] (1 - p_1) \]

\[ z F_1 [Sm + b_1, -dm, m - dm + a_1 + Sm + b_1, 1 - p_1] \alpha F_1 \]

\[ z F_1 [Sm + b_2, \cdots dm, n - dm + a_2 + Sm - Sm + b_2, 1 - p_2] \alpha F_2 h_1^{-1} (X) \]  \( (39) \)

\[ p_2^n = \left[ k \sum_{m=1}^{n-1} \Gamma(m - dm + a_1)I(m - dm + a_2) \Gamma(n - dm + a_1 + a_2) \right] (1 - p_2) \]

\[ z F_1 [Sm + b_2, -dm, n - dm + a_2 + Sm - Sm + b_2, 1 - p_2] \alpha F_2 dp_2 \]

\[ z F_1 [Sm + b_1, -dm, m - dm + a_1 + Sm + b_1, 1 - p_1] \alpha F_1 h_1^{-1} (X) \]  \( (40) \)

5. BAYES ESTIMATES USING INFORMATIVE AND NON INFORMATIVE PRIOR UNDER ASYMMETRIC LOSS FUNCTION

The Loss function \( L(a, d) \) provides a measure of the financial consequences arising from a wrong decision rule \( d \) to estimate an unknown quantity \( \alpha \). The choice of the appropriate loss function depends on financial considerations only, and is independent from the estimation procedure used. The use of symmetric loss function was found to be generally inappropriate, since for example, an over estimation of the reliability function is usually much more serious than an under estimation.

In this section, we derive Bayes estimator of change point \( m \) under different asymmetric loss function using both prior considerations explained in section 3. A useful asymmetric loss, known as the Linex loss function was introduced by Varian (1975). Under the assumption that the minimal loss at \( d \), the Linex loss function can be expressed as,

\[ L_4(a, d) = \exp [q_1 (d - a)] - q_1 (d - a) - 1, \]  \( q_1 \neq 0 \)

The sign of the shape parameter \( q_1 \) reflects the deviation of the asymmetry, \( q_1 > 0 \) if over estimation is more serious than under estimation, and vice-versa, and the magnitude of \( q_1 \) reflects the degree of asymmetry.

The posterior expectation of Linex loss function is:

\[ E_d [L_4(a, d)] = \exp (q_1 d) E_d [\exp (-q_1 a)] - q_1 (d - E_d [a]) - 1 \]

Where \( E_d [f(a)] \) denote the expectation of \( f(a) \) with respect to the posterior density of \( g(a|x) \). The Bayes estimate \( \alpha^* \) is the value of \( d \) that minimizes \( E_d [L_4(a, d)] \)

\[ \alpha^* = -\frac{1}{q_1} \ln [E_d [\exp (-q_1 a)] \]

Provided that \( E_d [\exp (-q_1 a)] \) exists and finite.

Minimizing expected loss function \( E_m [L_4(m, d)] \) and using posterior distribution (20) and (34) we get the bayes estimates of \( m \), using Linex loss function as,

\[ m_L^* = -\frac{1}{q_1} \ln \left[ \sum_{m=1}^{n-1} e^{-m q} l_1 (m) l_2 (m) \right] \]

\[ m_{L}^{**} = -\frac{1}{q_1} \ln \left[ \sum_{m=1}^{n-1} e^{-m q} l_2 (m) l_4 (m) \right] \]

Minimizing expected loss function \( E_{\theta_1} [L_4(\theta_1, d)] \) and using posterior distribution (15) and (30) we get the bayes estimates of \( \theta_1 \), using Linex loss function as
\[ \theta_{1L}^{*} = -\frac{1}{q_1} \ln \left[ \int_{0}^{1} (1 - \theta_1)^{m+a-1} \theta_1^{m+b_1-1} e^{-\theta_1 q_1} \right] \]

\[ 2F_1 \left[ c_1, -\left( -\frac{\theta_1}{\theta_1 - 1} \right) \frac{\partial}{\partial \theta_1} h_1^{-1}(X) \right] \]

Minimizing expected loss function \( E_{\theta_2} [L_4 (\theta_2, d)] \) and using posterior distribution (16) and (31), we get the bayes estimates of \( \theta_2 \), using Linex loss function as

\[ \theta_{2L}^{*} = -\frac{1}{q_1} \ln \left[ \int_{0}^{1} (1 - \theta_2)^{n-m-1} \theta_2^{n-Sm-b_2-1} e^{-\theta_2 q_1} \right] \]

Minimizing expected loss function \( E_p [L_4 (p_1, d)] \) and using posterior distribution (18) and (32), we get the bayes estimates of \( p_1 \), using Linex loss function as

\[ p_{1L}^{*} = -\frac{1}{q_1} \ln \left[ \int_{0}^{1} (1 - p_1)^{n-m-dm-1} p_1^{c_1-1} e^{-p_1 q_1} \right] \]

\[ 2F_1 \left[ c_1, -\left( -\frac{\theta_1}{\theta_1 - 1} \right) \frac{\partial}{\partial \theta_1} h_1^{-1}(X) \right] \]
\[ \int_{0}^{1} (1 - p_2)^{n-m-dn+dm-1} \]

\[ _2F_1 [Sn - Sm, -dn + dm, n - m - dn + dm + Sn - Sm, 1 - p_2] \phi_2 h_2^{-1}(X) \]  

Minimizing expected loss function \( E_{P_2} [L_4(p_2, d)] \) and using posterior distribution (19) and (33), we get the Bayes estimates of \( p_2 \), using Linex loss function as

\[ p_{2L}^* = -\frac{1}{q_1} \ln \left[ k \sum_{m=1}^{n-1} \Gamma(m - dm + a_1) \Gamma(Sm + b_1) \Gamma(n - m - dn + dm + a_2) \right] \]

\[ \Gamma(Sn - Sm + b_2) \int_{0}^{1} (1 - p_2)^{n-m-dn+dm+d_2-1} p_2 c_2^{-1} e^{-p_2 q_1} \]

\[ _2F_1 [Sn - Sm + b_2, -dn + dm, n - m - dn + dm + a_2 + Sn - Sm + b_2, 1 - p_2] dp_2 \]

\[ \int_{0}^{1} \{(1 - p_1)^{m-dm+d_1-1} p_1 c_1^{-1} \}

\[ _2F_1 [Sm + b_1, -dm, m - dm + a_1 + Sm + b_1, 1 - p_1] \phi_1 h_1^{-1}(X) \]  

\[ p_{2L}^* = -\frac{1}{q_1} \ln \left[ k_1 \sum_{m=1}^{n-1} \Gamma(m - dm) \Gamma(Sm) \Gamma(n - m - dn + dm) \Gamma(Sn - Sm) \right] \]

\[ \int_{0}^{1} (1 - p_2)^{n-m-dn+dm-1} _2F_1 [Sm, -dm, m - dm + Sm, 1 - p_2] e^{-p_2 q_1} dp_2 \]

\[ \int_{0}^{1} (1 - p_1)^{m-dm-1} _2F_1 [Sm, -dn + dm, n - m - dn + dm + Sn - Sm, 1 - p_2] \phi_1 h_2^{-1}(X) \]  

Another loss function, called General Entropy Loss function (GEL), proposed by Calabria and Pulcini (1994) is given by,

\[ L_5 (\alpha, d) = \left( d/\alpha \right)^{q_3} - q_3 \ln(d/\alpha) - 1. \]

The Bayes estimate \( \alpha_E^* \) is the value of \( d \) that minimizes \( E_d [L_5(\alpha, d)] \):

\[ \alpha_E^* = \left[ \left[ E_d (\alpha^{-q_3}) \right]^{-1} \right]^{q_3}. \]

Provided that \( E_d (\alpha^{-q_3}) \) exists and is finite.

Minimizing expectation \( E_m [L_5(m, d)] \) and using posterior distributions (20) and (34), we get Bayes estimate \( m_E^* \), \( m_E^* \), as,

\[ m_E = \left[ E (m^{-q_3}) \right]^{-\frac{1}{q_3}} \]

\[ m_E^* = \left[ E (m^{-q_3}) \right]^{-\frac{1}{q_3}} \]

\[ \int_{0}^{1} (1 - \theta_1)^{n-m-a_1-1} \theta_1^{Sm+b_1-q_3-1} _2F_1 [c_1, -dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}] d\theta_1 \]

\[ (1 - \theta_1)^{n-m+a_2-1} \theta_2^{Sm-Sm+b_2-1} \]

\[ \int_{0}^{1} (1 - \theta_2)^{n-m-a_2-1} \theta_2 h_2^{-1}(X) \]  

\[ \int_{0}^{1} (1 - \theta_1)^{m-1} \theta_1^{Sm-q_3-1} \]

\[ _2F_1 [-dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] \theta_1 \]

\[ \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 \Gamma(m - dm + d_1) \Gamma(n - m - dn + dm + d_2) \]

\[ \frac{\sum_{m=1}^{n-1} m^{-3} I_1(m) I_2(m)}{\sum_{m=1}^{n-1} I_1(m) I_2(m)} \]

\[ \int_{0}^{1} \{(1 - p_1)^{m-dm+d_1-1} p_1 c_1^{-1} \}

\[ \frac{\sum_{m=1}^{n-1} m^{-3} I_2(m) I_4(m)}{\sum_{m=1}^{n-1} I_2(m) I_4(m)} \]  

minimizing expectation \( E_{\theta_1} [L_5(\theta_1, d)] \) and using posterior distributions (15) and (30), we get Bayes estimate of \( \theta_1 \), using General Entropy loss function as

\[ \theta_{1E}^* = \left[ k_1 \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 \Gamma(m - dm + d_1) \Gamma(n - m - dn + dm + d_2) \right] \]

\[ \int_{0}^{1} (1 - \theta_1)^{n-m-a_1-1} \theta_1^{Sm+b_1-q_3-1} _2F_1 [c_1, -dm, c_1 + m - dm + d_1, \frac{\theta_1}{\theta_1 - 1}] d\theta_1 \]

\[ \int_{0}^{1} (1 - \theta_2)^{n-m-a_2-1} \theta_2^{Sm-Sm+b_2-1} \]

\[ _2F_1 [c_2, -(dn - dm), c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1}] \theta_2 h_2^{-1}(X) \]  

\[ \int_{0}^{1} (1 - \theta_1)^{m-1} \theta_1^{Sm-q_3-1} \]

\[ _2F_1 [-dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] \theta_1 \]  

\[ \int_{0}^{1} (1 - \theta_1)^{m-1} \theta_1^{Sm-q_3-1} \]

\[ _2F_1 [1, -dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] \theta_1 \]  

\[ \int_{0}^{1} (1 - \theta_1)^{m-1} \theta_1^{Sm-q_3-1} \]  

\[ _2F_1 [1, -dm, 1 - dm + m, \frac{\theta_1}{\theta_1 - 1}] \theta_1 \]  

62
\[ 2F_1 \left[ 1, dm - dn, 1 + dm - dn - m + n, \frac{\theta_2}{\theta_2 - 1} \right] \frac{\partial \theta_2 h_2^{-1}(X)}{\partial \theta_2} \left[ 1 \right] (54) \]

minimizing expectation \( [E_{\theta_2} [L_5 (\theta_2, d)] \) and using posterior distributions (16) and (31), we get Bayes estimate of \( \theta_2 \) using General Entropy loss function as,

\[
\theta_{2E}^* = \left[ k \sum_{m=1}^{n-1} \Gamma c_1 \Gamma c_2 (m - dm + d_1) \Gamma (n - m - dn + dm + d_2) \right. \\
\left. \int_0^1 (1 - \theta_2)^{m + a_2 - 1} \theta_2^{sn - sm - q_3 - 1} \right] \frac{1}{q_3} (55) \\

\[ 2F_1 \left[ c_1, -dn - dm, c_2 + n - m - dn + dm + d_2, \frac{\theta_2}{\theta_2 - 1} \right] \frac{\partial \theta_2 h_2^{-1}(X)}{\partial \theta_2} \left[ 1 \right] (55) \]

minimizing expectation \( [E_{p_1} [L_5 (p_1, d)] \) and using posterior distributions (18) and (32) we get Bayes estimate of \( p_1 \) using General Entropy loss function as,

\[
p_{1E}^* = \left[ k \sum_{m=1}^{n-1} \Gamma (m - dm + a_1) \Gamma (sm + b_1) \Gamma (n - m - dn + dm + a_2) \right. \\
\left. \int_0^1 (1 - p_1)^{m - dm + a_2 - d_1 - q_3 - 1} p_1^{a_2 - q_3 - 1} \right] \frac{1}{q_3} (57) \\

\[ 2F_1 \left[ sm + b_1, -dn + dm, m - dm + a_1 + sm + b_1, 1 - p_1 \right] \frac{\partial p_1 h_2^{-1}(X)}{\partial p_1} \left[ 1 \right] (57) \]

minimizing expectation \( [E_{p_2} [L_5 (p_2, d)] \) and using posterior distributions (19) and (33) we get Bayes estimate of \( p_2 \) using General Entropy loss function as,

\[
p_{2E}^* = \left[ k \sum_{m=1}^{n-1} \Gamma (m - dm + a_1) \Gamma (sm + b_1) \Gamma (n - m - dn + dm + a_2) \right. \\
\left. \Gamma (sm - sm + b_2) \int_0^1 (1 - p_2)^{m - dm + a_2 - d_2 - q_3 - 1} p_2^{a_2 - q_3 - 1} \right] \frac{1}{q_3} (58) \\

\[ 2F_1 \left[ sm + b_2, -dn + dm, m - dm + a_2 + sm - sm + b_2, 1 - p_2 \right] \frac{\partial p_2 h_2^{-1}(X)}{\partial p_2} \left[ 1 \right] (58) \]
\[ \int_0^1 \left\{ (1 - p_1)^{m - dm + 1} - 1 - p_1 \right\} d\phi_1 h_1^{-1}(X) \right] \frac{1}{\sigma_3} \]  

(59)

\[ p_2^* = \left[ k_1 \sum_{m=1}^{n-1} \Gamma(m - dm) \Gamma(Sm) \Gamma(n - m - dn + dm) \Gamma(Sn - Sm) \right] \int_0^1 \left( (1 - p_2)^{n-m-dm+1} - p_2 - \frac{1}{p_2} \right) d\phi_2 \]

\[ \int_0^1 (1 - p_1)^{m-dm+1} p_1^{dm} \binom{2}{1} \{ Sm, -dn + dm, n - m + dm + Sn - Sm, 1 - p_1 \} \phi_1 h_1^{-1}(X) \right] \frac{1}{\sigma_3} \]

(60)

6. NUMERICAL STUDY

We have generated 20 random observations from the proposed change point model explained in section 2. The first 10 observation from ZIGP with \( \theta_1 = 0.5 \) and, \( p_1 = 0.6 \) and the next 10 observations are from the same distribution with parameters \( \theta_2 = 0.7 \) and \( p_2 = 0.8 \). The \( p_1 \) and \( p_2 \) are themselves were random observations form Beta distributions with mean \( \mu_1 = 0.5 \) and \( \mu_2 = 0.7 \) and standard deviation \( \sigma = 0.1 \) respectively, resulting in \( c_1 = 0.63 \), \( d_1 = 0.82 \), \( c_2 = 0.84 \) and \( d_2 = 0.56 \). The observations are given in table-1.

<table>
<thead>
<tr>
<th>Table-1</th>
<th>Generated Observations From Proposed Change Point Model Of Zero Inflated Geometric Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xi</td>
<td>0 1 2 3 4 5 6 7 8 9 10</td>
</tr>
<tr>
<td>Xi</td>
<td>0 2 0 2 7 0 1 1 2 0 2</td>
</tr>
</tbody>
</table>

We have calculated posterior means of proportions under the informative and non-informative priors using the results given in section 4. The results are shown in Table-2.

<table>
<thead>
<tr>
<th>Table-2</th>
<th>The value of Bayes estimates of proportions under SEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior</td>
<td>Bayes Estimates of Proportion</td>
</tr>
<tr>
<td></td>
<td>( p_1 )</td>
</tr>
<tr>
<td>Informative</td>
<td>0.60</td>
</tr>
<tr>
<td>Non-Informative</td>
<td>0.59</td>
</tr>
</tbody>
</table>

We have calculated posterior means of \( m, \theta_1 \) and \( \theta_2 \) under informative and non-informative prior using the results given in section 4. The results are shown in Table 3.

<table>
<thead>
<tr>
<th>Table-3</th>
<th>Bayes Estimate of ( m, \theta_1 ) and ( \theta_2 ) under SEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior</td>
<td>Bayes Estimates of ( m ) (Posterior Mean)</td>
</tr>
<tr>
<td></td>
<td>( \theta_1^* )</td>
</tr>
<tr>
<td>Informative</td>
<td>10</td>
</tr>
<tr>
<td>Non – Informative</td>
<td>10</td>
</tr>
</tbody>
</table>
We also compute the Bayes estimators of $p_1$ and $p_2$ for both informative and non-informative priors using (47), (48), (49), (50), (57), (58), (59) and (60) respectively for the data given in Table-1 and for different value of shape parameter $q_1$ and $q_3$. The results are shown in Table-4.

### Table-4

<table>
<thead>
<tr>
<th>Prior</th>
<th>Shape Parameter</th>
<th>Bayes Estimates of Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q_1$</td>
<td>$q_3$</td>
</tr>
<tr>
<td>Informative</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Non- Informative</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

We have calculated Bayes estimate of change point $m$ and of $\theta_1$ and $\theta_2$ under the informative under Linex and General entropy loss functions respectively using (41), (51) and (43), (45), (53), (55) for the data given in Table-1 and for different value of shape parameter $q_1$ and $q_3$. The results are shown in Table-5 and 6.

### Table 5.

The Bayes estimates using Linex Loss Function

<table>
<thead>
<tr>
<th>$q_1$</th>
<th>$m_L$</th>
<th>$\theta_1^L$</th>
<th>$\theta_2^L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informative Prior</td>
<td>0.09</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>9</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>-1.0</td>
<td>11</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>-2.0</td>
<td>12</td>
<td>0.8</td>
</tr>
</tbody>
</table>

### Table 6

The Bayes estimates using General Entropy Loss Function

<table>
<thead>
<tr>
<th>$q_3$</th>
<th>$m_E$</th>
<th>$\theta_1^E$</th>
<th>$\theta_2^E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informative Prior</td>
<td>0.09</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>8</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>7</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>-1.0</td>
<td>12</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>-2.0</td>
<td>13</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 6 shows that for small values of $|q|$, $q_1=0.09$, 0.1, 0.2 Linex loss function is almost symmetric and nearly quadratic and the values of the Bayes estimate under such a loss is not far from the posterior mean. Table 5 also shows that, for $q_1 = 1.5, 1.2$, Bayes estimate are less than actual value of $m=10$.

For $q_3= q_3 = -1, -2$, Bayes estimates are quite large than actual value $m=10$. It can be seen from Table 5 and 6 that the negative sign of shape parameter of loss function reflecting underestimation is more serious than overestimation. Thus problem of underestimation can be solved by taking the value of shape parameters of Linex and General Entropy loss function negative.

Table 6 shows that, for small values of $|q|$, $q_3=0.09$, 0.2, 0.1 General Entropy loss function, the values of the Bayes estimate under a loss is not far from the posterior mean. Table 6 also shows that, for $q_3=1.5, 1.2$, Bayes estimates are less than actual value of $m=10$.

It can be seen Table 5 and 6 that positive sign of shape parameter of loss functions reflecting overestimation is more serious than under estimation. Thus problem of overestimation can be solved by taking the value of shape parameter of Linex and General Entropy loss function positive and high.
7. SIMULATION STUDY
In section 4 and 5, we have obtained Bayes estimates of \( m \) and proportions \( p_1 \) and \( p_2 \) on the basis of the generated data given in Table-1 for given values of parameters. To justify the results, we have generated 10,000 different random samples with \( m=10, n=20, p_1=0.6, p_2=0.8, \theta_1=0.5, \theta_2=0.7 \) and obtained the frequency distributions of posterior mean, median of \( m \), \( m_L^* \), \( m_E^* \) with the correct prior consideration. The result is shown in Table-7 and 8. The value of shape parameter of the general entropy loss and Linex loss used in simulation study for change point is taken as 0.1. We have also simulated several mixed samples as explained in section 2 with \( p_1 = 0.5, 0.6, 0.7; p_2 = 0.5, 0.8, 0.9 \) and \( \theta_1 = 0.15, 0.11, 0.10; \theta_2 = 0.55, 0.45, 0.35 \). For each \( p_1, p_2, \theta_1 \) and \( \theta_2 \), 1000 pseudo random samples with \( m=10 \) and \( n=20 \) have been simulated and Bayes estimators of change point \( m \) using \( q_2 = 0.9 \) has been computed for same value of \( a, b, c_i \) and \( d_i \) \( i=1, 2 \) for different prior means \( \mu_1 \) and \( \mu_2 \).

<table>
<thead>
<tr>
<th>Bayes estimate</th>
<th>% Frequency for</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>01-08</td>
</tr>
<tr>
<td>Posterior mean</td>
<td>17</td>
</tr>
<tr>
<td>Posterior median</td>
<td>17</td>
</tr>
<tr>
<td>Posterior mode</td>
<td>13</td>
</tr>
<tr>
<td>( m_L^* )</td>
<td>22</td>
</tr>
<tr>
<td>( m_E^* )</td>
<td>16</td>
</tr>
</tbody>
</table>

8. REFERENCES