A THEORETICAL MODEL OF STABLE DARK ENERGY STARS

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ABSTRACT

In this paper we present a relativistic static and spherically symmetric dark energy stellar configuration consisting of an interior anisotropic fluid governed by a dark energy like equation of state matched to an exterior Schwarzschild vacuum solution through a thin shell. A mass function depending on an adjustable parameter is imposed to solve the Einstein field equations. Physical properties and stability of the obtained new family of solutions is further explored.

Keywords: dark energy, equation of state, field equations, mass function.

1. INTRODUCTION

Recent astrophysical observations have confirmed that the Universe shows an accelerated cosmic expansion [1]. Evidence of this expansion has been shown independently from measurements of supernovae of type Ia and from microwave background radiation [2]. It is proposed that this cosmological behavior is caused by a hypothetical dark energy, a cosmic fluid parameterized by an equation of state $\omega = p/\rho < -1/3$ where $p$ is the spatially homogeneous pressure and $\rho$ the dark energy density [1,2,3]. The range for which $\omega < -1$ has been denoted phantom energy, and possesses peculiar properties, such as negative temperatures and the energy density increases to infinity in a finite time, resulting in a big rip [2,3,4]. It also provides a natural scenario for the existence of exotic geometries such as wormholes [5,6].

Relatively to the issue of gravitational collapse, before the mid-1960s, the object now known as a black hole, was referred to as collapsed star [4]. Oppenheimer and Snyder [7], in 1939, provided the first insights of the gravitational collapse into a black hole, however, it was only in 1965 that marked an era of intensive research into black hole physics. In this line of research, it is interesting to note that a new final state of gravitational collapse has been proposed by Mazur and Mottola [8]. This model, denoted as a gravastars, consists of a compact object, whose fluid is governed by an equation of state given by $p = -\rho$. When these types of fluids are taken to a star model scenario and gravitational collapse, the anisotropy may play a very important roll [9]. Cattoen, Faber and Visser [10] and Chan, da Silva and Villas da Rocha [11] have shown that gravastars models must exhibit anisotropic pressures. In a simplified model of the Mazur – Mottola picture, Visser and Wiltshire [12] constructed a model by matching an interior solution with an equation of state $p = -\rho$, to an exterior Schwarzschild solution with a junction interface. Lobo [3], motivated by the picture of gravastars, has proposed a model of stars consisting of dark energy. The analysis in his work is based on the dark energy limits for the parameter $\omega$ of the equation of state, $p = \omega \rho$ with $-1 \leq \omega \leq -1/3$. This interval of values comes from the Friedmann cosmological models which assume isotropic pressures $p$. For the particular case $\omega = -1$ reduces to the Visser-Wiltshire model.

Chan, da Silva and Villas da Rocha [9] proposed a generalization of these limits for the case of anisotropic fluids and this generalization comes directly from the strong energy condition $\rho + p_r + 2p_t \geq 0$, $\rho + p_r \geq 0$ and $\rho + p_t \geq 0$.

The notion of dark energy is that of a homogeneously distributed cosmic fluid and when extended to inhomogeneous spherically symmetric spacetimes, the pressure appearing in the equation of state is now a negative radial pressure, and the transverse pressure is then determined via the field equations [2,3]. The generalization of the gravastars picture with the inclusion of an interior solution governed by the equation of state $p = \omega_0 \rho$ with $\omega_0 < -1/3$, will be denoted by dark energy star in agreement with Chapline [12]. Lobo [3] explored several configurations, by imposing specific choices for the mass function and studied the dynamical stability of these models by applying the general stability formalism developed by Lobo and Crawford [13]. Chan, da Silva and Villas da Rocha [14] propose the mass function is a natural consequence of the Einstein’s field equations and considered a core with a homogeneous energy density, described by the Lobo’s first solution [3].

It is known that dark energy exerts a repulsive force on its outskirts [2,3]. With the aim to study how dark energy affects the gravitational collapse, we propose a model consisting of a nucleus (core) of dark energy described by a spherical anisotropy distribution of matter joined to the exterior Schwarzschild space through a thin shell.
Within the core the positive energy density associated to the given mass function, exhibits a positive gradient. We impose the energy conditions related to dark energy to ensure gravitational repulsion characteristics of dark energy and evaluate the model behavior and instability.

The inner core is characterized by the equation of state \( p_r = \omega \rho \), where \( p_r \) is the radial pressure while the transverse pressure \( p_t \) is found from field equations. A new mass function depending of an adjustable parameter is introduced in order to solve the field equations and obtain several configurations. This mass function represents an increase of the energy density in the star interior in contrast with usual models. This radial dependence of the mass occurs when modeling the phenomenon of repulsive vacuum gravitation, related to inflationary-universe models [15]. In the electromagnetic mass model of Tiwari et. al. [16] obeying a \( p=-\rho \) equation of state, the gravitational mass grows as \( r^4 \) inside the star. It seems interesting to propose a model with this radial dependence, considering the fluid trapped within a shell and to evaluate its physical properties and stability, to model a star of anisotropic fluid in a dark energy scenario, where gravitational repulsion is expected to occur. Wyman [17] proposed a similar solution, where the density distribution is given by \( \rho = ar^n \), where \( a \) and \( n \) are constants. Also Bayin [18] have considered this type of energy distribution, which may be used where density inversion occurs. Moreover, this mass function could describe portions of stars. We assume that the denomination dark energy is applied to fluids which violate only the strong energy condition. Following closely the stability formalism of Lobo and Crawford [13], we analyze the stability of these models. It is found that the stability regions increase for a decreasing dark energy parameter \( \omega \) and an increasing of the parameter associated to the mass function, the stability regions appear in zones of higher gravitational potential in contrast with the model of Lobo [3] where the stability regions are very insensitive to variations in \( \omega \) and gravitational potential.

This paper is organized as follows. In the Section II, we present the structure equations of dark energy stellar models. In Section III, specific model is analyzed by imposing particular choice for the new mass function. In Section IV, the stability analysis is briefly outlined. In Section V, we conclude.

2. DARK ENERGY STARS: THE FIELD EQUATIONS

We shall model dark energy stars by considering a spherically symmetric distribution of matter consisting of an anisotropic fluid governed by an equation of state, \( p = \omega \rho \) with \( \omega < 0 \), joined to the vacuum Schwarzschild exterior space through a thin spherical shell of matter with junction radius \( a \) [2,3].

The interior static and spherically symmetric spacetime is described by the following metric, in curvature coordinates [3]

\[
ds^2 = -e^{2\phi(r)}dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2(d\theta^2 + \sin^2 d\phi^2)
\]

(1)

where \( \phi(r) \) and \( m(r) \) are arbitrary functions of the radial coordinate, \( r \). The function \( m(r) \) is the quasi-local mass, and it is denoted as the mass function. \( \phi(r) \) is the red shift function defined as

\[
\phi(r) = \int_r^\infty g(\tilde{r})d\tilde{r}
\]

(2)

The Einstein field equation, \( G_{\mu\nu} = 8\pi T_{\mu\nu} \), where \( G_{\mu\nu} \) is the Einstein tensor and \( T_{\mu\nu} \) the stress-energy tensor, provides the following relationships:

\[
m' = 4\pi r^2 \rho
\]

(3)

\[
g(r) = \frac{m + 4\pi r^3 p_r}{r(r - 2m)}
\]

(4)
\[
p'_r = - \left( \frac{\rho + p_r}{r(r-2m)} \right) (m + 4\pi a^3 p_r) + \frac{2}{r} (p_r - p_t) \tag{5}\]

where the prime denotes differentiation with respect to the radial coordinate, \( r \). \( \rho(r) \) is the energy density, \( p_r(r) \) is the radial pressure and \( p_t(r) \) is the transverse pressure. Equation (5) corresponds to the anisotropic pressure Tolman-Oppenheimer-Volkof (TOV) equation.

Now, using the dark energy equation of state, \( p_r = \omega \rho \), and taking Eqs. (3) and (4) into account, we have the following relationship

\[
g(r) = \frac{m + \omega a m'}{r(r-2m)} \tag{6}\]

Using the dark energy equation of state \( p_r = \omega \rho \), Eq. (5) in terms of the principal pressures, takes the form

\[
p'_r = -p_r \left( \frac{\omega + 1}{\omega} \right) \left( \frac{m + \omega a m'}{r(r-2m)} \right) + \frac{2}{r} (p_r - p_t) \tag{7}\]

which taking into account Eq. (3), may be expressed in the following equivalent form

\[
\Delta = \frac{\omega}{8\pi a^2} \left[ m'' r - 2m' + \left( \frac{1 + \omega}{\omega} \right) m' r g \right] \tag{8}\]

\( \Delta = p_t - p_r \) is denoted as the anisotropic factor, as it is a measure of the pressure anisotropy of the fluid comprising the dark energy star. \( \Delta = 0 \) corresponds to the particular case of an isotropic pressure dark energy star [3].

Now we use the Israel formalism [19] to match the interior solution above to the Schwarzschild exterior metric. The surface stresses for this particular case takes the form

\[
\sigma = -\frac{1}{4\pi a} \sqrt{\frac{1 - 2m}{a} + \dot{a}^2} - \sqrt{\frac{1 - 2m}{a} + \dot{a}^2} \tag{9}\]

\[
P = \frac{1}{8\pi a} \left[ \frac{1}{\sqrt{\frac{1 - 2m}{a} + \dot{a}^2}} - \frac{1 + \omega m' - \frac{m}{a} + \dot{a}^2 + \dot{a} \dot{a} + \dot{a}^2 m'(1 + \omega)}{\frac{1 - 2m}{a}} \right] \tag{10}\]

where \( \sigma \) and \( P \) are the surface energy density and the surface pressure, respectively. The surface mass of the thin shell is given \( m_s = 4\pi a^2 \sigma \). By rearranging Eq. (9), evaluated at a static solution \( a_0 \), one obtains the total mass of the dark energy star, given by

\[
M = m(a_0) + m_s(a_0) \left[ \sqrt{1 - \frac{2m(a_0)}{a_0}} - \frac{m_s(a_0)}{2a_0} \right] \tag{11}\]
3. THE MODEL
Recently Chaisi and Maharaj [20] obtained a new class of exact solutions from the Einstein’s field equations for anisotropic spheres. They took the energy density as:

$$\rho = \frac{j}{r^2} + k + lr^2$$  \hspace{1cm} (12)

Where j, k and l are constants. This class contains the constant density model of Maharaj and Maartens [21] and the variable density model of Gokhroo and Mehra [22] for anisotropic spheres as special cases. The expression for the mass function with the particular energy density (12) is

$$m(r) = \frac{r}{2} \left( j + \frac{k}{3} r^2 + \frac{l}{5} r^4 \right)$$  \hspace{1cm} (13)

Now, in order to find an anisotropic solution that allows to model a dark energy star, presenting a repulsive gravitational force, inspired in the Tiwari et al. solutions [16], we propose a mass function depending of a adjustable parameter by using (13). With j = k = 0 the mass function becomes

$$m(r) = \frac{lr^5}{10}$$  \hspace{1cm} (14)

We impose the conditions expected to hold for a dark energy scenario and evaluate the obtained results. The spacetime metric for this solution is given by

$$ds^2 = -\left(1 - \frac{lr^4}{5}\right)^{-(\frac{5\omega+1}{4})} dt^2 + \frac{dr^2}{1 - \frac{lr^4}{5}} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)$$  \hspace{1cm} (15)

Note that with the new mass function the spacetime metric (15) does not diverge in the origin r = 0. For this mass function, the violation of strong energy condition furnishes us

$$5m(r)\omega^2 + \left[5r - 4m(r)\right]\omega + r - m(r) \leq 0$$  \hspace{1cm} (16)

which is always satisfied for $\omega < -1/5$ and $\omega > 1 - \frac{r}{m(r)}$ where $m(r) = \frac{lr^5}{10}$ and r are the mass and the radial coordinate, respectively.

This metric can be rewritten as

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right)^{-(\frac{5\omega+1}{4})} dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)$$  \hspace{1cm} (17)

The stress-energy tensor components are given by $p_r = \omega r^2$ and

$$p_t = 2\omega lr^2 \left[ 1 + \frac{(\omega + 1)(\omega + 1/5)lr^4}{8\omega (1 - lr^4/5)} \right]$$  \hspace{1cm} (18)

The strong energy condition vs the radial coordinate is plotted in Fig. 1 for different values of $\omega$ with a fixed value of the surface gravitational potential M/a=0.38 and l=0.1. For $\omega < -1/5$ it takes negative values indicating a violation of strong energy condition. Figures 2 and 3 show, for a fixed value of the surface gravitational potential M/a and fixed values of l, the variation of the ratio of the tangential pressure to the energy density within
the sphere for different values of $\omega$. It is seen how the dominant energy condition is fulfilled by the different values of $l$ and $\omega$.

Figure 4 shows the variation of radial pressure $p_r$ vs the radial coordinate for different values of $\omega$ for the values of $l=0.1$ and $M/a = 0.38$. Observe $p_r$ diminish for decreasing dark energy parameter $\omega$. In Fig. 5, for a value of $M/a = 0.38$ and $l = 0.02; 0.06; 0.1$, it is shown the dependence of the energy density $\rho$ with $x=r/a$. Observe $\rho$ increases toward the exterior of the sphere for an enlargement of the values of $l$.

To see how the $l$ parameter values modify the space time metric, we fix the value of $M/a$ and $\omega$ and take different values of $l$. In Fig. 6, it is shown the metric function for the values of $l = 0.02; 0.06$ and $0.1$ for a gravitational surface potential $M/a=0.38$ and $\omega=-0.5$, respectively.

Notice that for the larger values of $l$, the metric function diminishes, as it is expected, due to the value $\omega=-0.5$. It is interesting to remark that this value is in the range of $\omega$ that violates the strong energy condition, $-1.1 < \omega < -1/5$, as it is shown in figure 1.

**Fig. 1.** The strong energy condition vs the radial coordinate with $M/a=0.38$ and $l=0.1$ for different values of $\omega$. Lines labeled a,b,c and d correspond to the values of $\omega = -0.1, -0.15, -0.25$ and $-0.4$, respectively.
Fig. 2. Variation of the ratio of the tangential pressure to the energy density within the sphere for values of surface gravitational potential of $M/a = 0.26$ and $0.34$ and values of $l = 0.02$ and $0.06$. (a) gravitational potential of $M/a=0.26$ and $l=0.02$. (b) gravitational potential of $M/a=0.34$ and $l=0.06$. Lines labeled a, b, c and d correspond to $\omega = -0.5$, $-0.9$, $-1.3$ and $-1.4$. 

**Fig. 2.** Variation of the ratio of the tangential pressure to the energy density within the sphere for values of surface gravitational potential of $M/a = 0.26$ and $0.34$ and values of $l = 0.02$ and $0.06$. (a) gravitational potential of $M/a=0.26$ and $l=0.02$. (b) gravitational potential of $M/a=0.34$ and $l=0.06$. Lines labeled a, b, c and d correspond to $\omega = -0.5$, $-0.9$, $-1.3$ and $-1.4$. 

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Fig. 3. Variation of the ratio of the tangential pressure to the energy density within the sphere for values of surface gravitational potential of $M/a = 0.38$ and $0.40$ and value of $l = 0.1$. a gravitational potential of $M/a=0.38$. Point line, dashed line, point dashed line, thick solid line, pont dashed long line and two point dashed line correspond to $\omega = -0.5, -0.7, -0.9, -1.1, -1.2$ and $1.3$ b gravitational potential of $M/a = 0.4$. Dashed line, two point dashed line, thick dashed line, thick solid line and little dashed line correspond to $\omega = -0.5, -0.9, -1.3, -1.5, -1.6$ and $-1.7$. 
Fig. 4. Variation of radial pressure vs radial coordinate for different values $\omega$ for values of $l=0.1$ and $M/a = 0.38$. Lines labeled a, b, c and d correspond to $\omega = -0.1$, -0.15, -0.3 and -0.5.

Fig. 5. Dependence of the energy density with radial coordinate with $M/a = 0.38$. Lines labeled a, b and c correspond to $l = 0.02$, 0.06 and 0.1 respectively.
Fig. 6. Variation of metric function $e^{2\phi}$ of radial coordinate for different values of $l$ for a gravitational potential $M/a = 0.38$ and $\mathcal{O} = -0.5$. Lines labeled a, b and c correspond to $l = 0.02$, 0.06 and 0.1, respectively.

4. STABILITY REGIONS

We follow closely the Lobo and Crawford’s [13] approach to analyze stability of wormhole. They included the momentum and flux term in the conservation identity, deduced from the Lanczos equation [23] and the ADM constraint, into the Ishak and Lake analysis [24], to deduce the master equation which governs the stable equilibrium. We shall describe the approach briefly, details can be found in Lobo and Crawford [13] and reference there in.

The equation of motion of the thin shell is obtained from (9) and the resulting potential $V(a)$ is linearized by a Taylor expansion around the equilibrium radius $a_0$ of the static solution, to the second order

$$a^2 + V(a) = 0$$

(19)

With $V(a)$ defined as:

$$V(a) = \frac{1}{2} \left( 1 - \frac{2m}{a} + \frac{1 - 2M}{a} \right) - \frac{1}{2} \frac{m_s}{m} \left( 1 - \frac{2m}{a} - \frac{2M}{a} \right)^2$$

(20)

Next, one imposes $V(a_0)=0$ and $V'(a_0)=0$, therefore the solution will be stable if and only if $V(a)$ has a local minimum at $a_0$ and $V''(a_0)>0$ is verified.

$$V''(a) = \frac{1}{2} \left( 1 - \frac{2m}{a} + \frac{1 - 2M}{a} \right) - 2 \frac{m_s}{m} \left( 1 - \frac{2m}{a} - \frac{2M}{a} \right)^2$$

(21)

The latter stability condition may be written as

$$\left( \frac{m_s}{2a} \right)^2 < \Psi - \Gamma^2$$

(22)

Where $\Gamma$ is defined from the condition of $V'(a_0)=0$ as

$$\Gamma = \left( \frac{a_0}{m_s} \right) \left( \frac{1}{2} \left( 1 - \frac{2m}{a} + \frac{1 - 2M}{a} \right) - \frac{1}{2} \frac{m_s}{m} \left( 1 - \frac{2m}{a} - \frac{2M}{a} \right)^2 \right)$$

(23)
And \( \Psi \) is given as
\[
\Psi = \frac{1}{2} \left[ \left( 1 - \frac{2m}{a} \right) + \left( 1 - \frac{2M}{a} \right) \right]^{''} \left( 1 - \frac{2M}{a} \right) \left( 1 - \frac{2m}{a} \right) \left( \frac{a}{m} \right)^{'} \left( 1 - \frac{2m}{a} \right) \left( 1 - \frac{2M}{a} \right) \left( \frac{a}{m} \right)^{'}^{2}
\]
(24)

To obtain an expression for \((m_s/2a)^{''}\) we shall make use of the conservation law [2,3] given by
\[
S^i_{\ j||} = \left[ T_{\mu \nu} e^{\mu (j)} \eta^\nu \right]^{'}
\]
(25)

Eq.(25) provides us with
\[
\sigma' = -\frac{2}{a}(\sigma + P) + \Xi
\]
(26)
where \( \Xi \), defined for notational convenience, is given by
\[
\Xi = -\frac{1}{4\pi a (a - 2m)} \sqrt{1 - \frac{2m}{a} + \hat{a}^2}
\]
(27)

Using \( m_s = 4\pi a^2 \sigma \) and taking into account the radial derivative of \( \sigma' \), Eq. (26) can be rearranged to provide the following relationship
\[
\left( \frac{m_s}{2a} \right)^{''} = \frac{\gamma - 4\pi \sigma' \eta}{a}
\]
(28)

With the parameter \( \eta \) defined as \( \eta = \frac{P}{\sigma'} \) and \( \gamma \) given by
\[
\gamma = \frac{4\pi}{a} (\sigma + P) + 2\pi a \Xi'
\]
(29)

Equations (23)-(29) in (22) are used to determine the stability regions of the respective solutions, where \( \eta \) is used as a parametrization of the stable equilibrium, so that there is no need to specify a surface equation of state [3].

Following Poisson and Visser [25] and Ishak and Lake [24], in equation (28) \( \sqrt{\eta} \) is defined as the speed of sound [2,3]. Then, taking into consideration the requirement that the speed of sound should not exceed the speed of light, the values of \( \eta \) are constrained to \( 0 < \eta \leq 1 \) on the surface layer. We shall impose a positive surface energy density, \( \sigma > 0 \), which implies \( m(a) < M \). Considering the new mass function chosen, which represents an increase in the energy density inside the star, Fig. 7 shows the stability region for a dark energy star for the values of \( \omega = -0.5 \) and \( \omega = -0.9 \) with \( l = 0.02 \) in a and b, respectively. The stability region are given by the plots a and b depicted below the surfaces. For this model \( \eta \) shows an increase when \( m/M \) increases, indicating that the stability regions below the surface is increased. The region below the surface shown corresponds to stable equilibrium. The stability regions are very sensitive to variations in \( \omega \); note that the stability regions increases for decreasing \( \omega \). In the range \( 0 \leq m/M \leq 0.2 \) stability configurations are increased to high values of \( m/M \) and decreases for low values of \( m/M \). In Figure 8 it is shown the stability regions for \( \omega = -0.5 \) and \( \omega = -0.9 \) with \( l = 0.06 \), respectively, for a range \( 0 \leq m/M \leq 0.1 \). As in Fig. 7, it shows that the stability regions increase when the parameter \( \omega \) decreases, indicating an increase in regions that are below the surface. In this case, the configurations of stability moving to regions of higher gravitational potential. Fig. 9 shows the same behavior as Figures 7 and 8. For this particular case \( l = 0.1 \), which implies that \( M/a > 0.37 \), it shows the appearance of regions of higher gravitational potential.

It is observed that growth of the parameter \( l \) allows stability regions where the gravitational potential at the surface is high. It has also been shown that stability regions increase below the surface when the parameter \( l \) grows. For example, in Fig. 9 the values of \( \eta \) increase when decreasing the parameter \( \omega \) of dark energy, but they are larger than the observed values of \( \eta \) in Figures 7 and 8. Indeed, in Fig. 9, the stability regions below the surfaces correspond to higher values of \( \eta \), at difference to Fig. 7 and 8 where stable configurations have lower values of \( \eta \).
Fig. 7. Plots of the stability regions for a dark energy star with the new mass function with $l = 0.02$. We have considered $\omega = -0.5$ and $\omega = -0.9$, in the first and second plots, respectively.
Fig. 8. Plots of the stability regions for a dark energy star with the new mass function with $l=0.06$. We have considered $\omega = -0.5$ and $\omega = -0.9$, in the first and second plots, respectively.
5. CONCLUSION
In this paper, a new model of dark energy star was explored, by considering a matching of an interior solution governed by the dark energy equation of state, $\omega = p/\rho < -1/3$, to an exterior Schwarzschild vacuum solution at a junction interface. We have analyzed a relativistic dark energy stellar configuration by imposing specific choice for

Fig. 9. Plots of the stability regions for a dark energy star with the new mass function for $l=0.1$. We have considered $\omega=-0.5$ and $\omega = -0.9$ in the first and second plots, respectively.
the mass function which represents an increase in energy density inside the star. In this dark energy model the pressure decreases with the radial coordinate contrary to the energy density.

In this model, the mass function depends on a parameter $l$. Thus, the gravitational potential grows when the parameter $l$ increases, this is shown in Figures 7, 8 and 9. For example, $l = 0.02$ correspond m/a $> 0.25$ ; $l = 0.06$, m/a $> 0.33$ and $l = 0.1$, m/a $> 0.37$.

To evaluate the stability region we followed closely Lobo and Crawford [13] and Poisson and Visser [25] where $\eta = P/\sigma$ is used as a parametrization of the stability of equilibrium. The values of the parameter $l$ modify the stability regions.

The values of the parameter $l$ modify the stability regions for different values of m/a. In Figures 7, 8 and 9, it is shown how the increasing of the parameter $l$ allows stability regions to appear for greater values of gravitational potential in contrast to the model of Lobo [3], where there are not large variations in the configurations of stability. Finally, we would like to emphasize that even though the mass function provides an increasing radial energy density, no physically accepted to model boson stars, we have intended here to model an anisotropic fluid trapped in a thin shell, in a dark energy scenario, where the gravity repulsion is expected. Energy densities with this radial dependence appear in the electromagnetic mass models of Tiwari et al.[16] where gravitational repulsion is present due to the strain of the vacuum because of vacuum polarization. It could be the case for anisotropic fluids, where the gradient of the tangential pressures keeps the equilibrium with the repulsive gravitational force. The existence of regions of stability for specific values of the parameters of the model have encouraged us to propose this model as an approach to describe a star in a dark energy scenario. As we mentioned before, also this results may be used to describe regions of a star where density inversion process were expected to occur.

6. REFERENCES

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