NUMERICAL COMPARISON OF PRICING OF EUROPEAN CALL OPTIONS FOR MEAN REVERTING PROCESSES

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ABSTRACT
We propose a change of probability measure that allows to find a partial differential equation for valuing European call options on processes mean reversion, whose solution is approximated numerically by Adomian decomposition method. To test the convergence of the method is presented an analytic function that is convergent with the terminal condition of the differential equation and consistent with payy-off of a European call option. We compare the numerical results obtained with alternative methods to value options.

Keywords: *Option pricing, Adomian decomposition method, measuring change, finite difference scheme, binomial trees, Monte Carlo Simulation.*

1. INTRODUCTION

Since the initial proposal of Black and Scholes (1973) have emerged many option pricing models more complex in which are explicitly incorporate mean reversion processes. For example, stochastic volatility models proposed by S. Heston (1993), S, Heston and S. Nandi (1997) and D. Nelson (1990) incorporate mean reversion processes for volatility modeling. Such processes have also been used to model the dynamic behavior of interest rates O. Vasicek (1977), M. Brennan and E. Schwartz (1980), J. Cox, J. Ingersoll and S. Ross (1985), K. Chan, F. Karolyi, F. Longstaff and A. Sanders (1992) and spot prices of some agricultural commodities H. Bessembinder et al. (1995), also there is clear evidence that some energetics S. Deng (1999) and A. Lavassani et al. (2001a), can be modeled with processes associated to models of mean reversion with constant parameters and even more sophisticated models that include jumps and functional parameters, deterministic or stochastic as described in D. Pilipovic (2007) and H. Geman and A. Roncoroni (2008).

In the literature you can find some alternative numerical methods for valuing options. For example, in M. Brennan and E.Schwartz (1977), G. Courtadon (1982a) and J. Hull and A. White (1990) studied the finite difference method, some Monte Carlo simulation studies can be consulted in M. Broadie and P. Glasserman (1996), a detailed description of the scheme with applications of binomial trees found in J. Cox , J. Ross and M. Rubinstein (1979), and A. Lavassani et al (2001 b) while in M. Bohner, and Y. Zheng (2009) and M. Bohner, F. Marin and S. Rodriguez (2013) proposed the method of decomposition of G. Adomian, (ADM), (1994). All these methods have one thing in common: they assume that the underlying asset follows a geometric Brownian motion. For the special case where the underlying asset follows a process of mean reversion, A. Lavassani et al (2001 b) has a binomial trees recombination process including proportional noise, and F. Marin (2010) proposes a multiplicative recombination generalized binomial trees that are included the of mean reversion processes with additive and proportional noise.

This paper presents a proposal for valuing options on models of mean reversion with proportional noise using method (ADM). Although this presentation focuses primarily on the model of M. Brennan and E. Schwartz (1980), that can be adapted to other models of mean reversion as described in O. Vasicek (1977) and J. Cox, J. Ingersoll and S. Ross (1985). It is proposed as a change in the underlying process to obtain a partial differential equation similar to the Black Scholes. To test the convergence of the method provides an approximate boundary condition consistent with payy-off of a European call option and compared the results with those obtained with finite difference methods, binomial trees and Monte Carlo simulation.

This paper is organized as follows. In the second section, we will describe the models of mean reversion of a single factor with constant parameters. The change of measure which allows a partial differential equation and a new stochastic differential equation to model the underlying asset is presented in section 3. In Section 4, we present the Adomian decomposition method applied to the problem of European call option pricing. A brief description of finite difference methods, binomial trees and Monte Carlo simulation for option valuation under mean reversion processes will be made in section 5. Section 6 will present the numerical results of the experimental method (ADM) compared to alternative methods.

2. MEAN REVERSION PROCESSES OF A SINGLE FACTOR

The unidimensional mean reversion processes with one factor with constant parameters are given by the general equation

$$dx_t = \alpha(L - x_t)dt + \sigma x_t^{\gamma} dB_t; \quad t \in [0;T] \tag{1}$$

defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}$, with initial condition $x_0 = x$, where $\alpha > 0, \sigma > 0, L \in \mathrm{R}$ and $\gamma \in \left[0, \frac{3}{2}\right]$ are constants and $\{B_t\}_{t\geq 0}$ is a Unidimensional standard Brownian motion defined on the same space.

The parameter α is known as reversion rate, σ is the parameter associated with the volatility, γ is the elasticity of variance and L is the level of average or trend reversal long run equilibrium, ie a finite time horizon, L plays the role of an attractor in the sense that where $x_t > L$ the trend term $\alpha(L - x_t) < 0$ and therefore x_t decreases and when $x_t < L$ a similar argument establishes that x_t grows.

The Equation (1) is known as CKLS model proposed by K. Chan, F. Karolyi, F. Longstaff and A. Sanders (1992) and is a generalization of the models of interest rate in the short term with constant parameters in which $\gamma = 0, 1/2, 1$ or $\gamma = 32$ with $\alpha = 0$.

Although this general model derived various interest rate models studied in the literature, our interest focuses primarily on the special case $\gamma = 1$. That is, the process has proportional noise and obtained a inhomogeneous linear stochastic differential equation given by

$$dx_t = \alpha(L - x_t)dt + \sigma x_t dB_t, \quad x_0 = x; \quad t \in [0;T] \tag{2}$$

which is used by M. Brennan and E. Schwartz (1980) to obtain a numerical model of convertible bond prices. This process is also used by G. Courtadon (1982b) in developing a model of option pricing on discount bonds.

3. CHANGE OF MEASURE

As the expected value of the discounted payment of a derivative under the subjective probability $\mathbb{P}$ would lead to arbitrage opportunities, it is necesary build a unique probability measure $\mathbb{P}^*$ equivalent to $\mathbb{P}$ such that the discounted price at the rate of risk-free interest r , is a martingale and the expected value of the derivative instrument discounted payment under the objective probability $\mathbb{P}^*$ present no arbitrage opportunities.

In M. Garman (1976), J, Cox, J. Ingersoll and S. Ross (1985) and J. Hull and A. White (1990) states that the value of the option $u(x,t)$ where the underlying asset follows a process given by equation (2) must satisfy the partial differential equation

$$\frac{\partial u}{\partial t} + [\alpha(L - x) - \lambda(x,t)x]\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\sigma^2 x^2 - ru = 0 \tag{3}$$

where $\lambda(x,t)$ represents the market risk for x, and r is the risk free rate. In S. Heston (1993) assumes that the market risk is proportional to volatility, ie, $\exists\lambda$ nonnegative constant such that $\lambda(x,t) = \lambda\sigma$.

Consequently, equation (3) is being

$$\frac{\partial u}{\partial t} + [\alpha(L - x) - \sigma\lambda x]\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\sigma^2 x^2 - ru = 0 \tag{4}$$

A European call option with strike price K and time to maturity T satisfies the equation (4) and the problem is complete, subject to the following boundary conditions

$$u(0,t) = 0; u(x,T) = Max\{x - K, 0\} \tag{5}$$

For Girsanov theorem (see Mao, X. (1997)),

$$dB_t^* = dB_t + \lambda(x,t)dt \tag{6}$$

$$\frac{d\mathbb{P}^*}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t |\lambda(x,u)|^2 du - \int_0^t \lambda(x,u)dB_u\right) \tag{7}$$

$$\lambda(x,t) = \frac{\alpha\left(\frac{L}{x}-1\right)-r}{\sigma} \tag{8}$$

Where $\mathbb{P}$ is the measure in the real world under the subjective probability $\mathbb{P}$ and $\{B_t^*\}_{0\le t\le T}$ is a $\mathbb{P}^*$ - Brownian motion. Under the objective probability $\mathbb{P}^*$, equations (2) and (4) are being respectively

$$dx_t = \beta(\theta - x_t)dt + \sigma x_t dB_t^* \tag{9}$$

$$\frac{\partial u}{\partial t} + \beta[\theta - x]\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\sigma^2 x^2 - ru = 0 \tag{10}$$

The modified parameters are $\beta = \alpha + \lambda$ and $\theta = \dfrac{\alpha L}{\alpha + \lambda}$;

where,
r : Is the interest rate risk free.
β : Reversion rate.
σ : Volatility of the underlying asset.
θ : Reversion level half or steady trend

4. ADOMIAN DECOMPOSITION METHOD

Following the results of G. Adomian (1994) and D. Lesnic (2006), consider the partial differential equations, linear time-dependent one-dimensional form

$$\sum_{n=0}^{N}\alpha_n(x,t)\frac{\partial^n u}{\partial t^n}(x,t) = \sum_{m=1}^{M}\phi_m(x,t)\frac{\partial^m u}{\partial x^m}(x,t) + f(x,t), (x,t) \in \Omega \subset \mathbb{R}^2, \tag{11}$$

where $\alpha_n, 0 \le n \le N$ y $\phi_m, 1 \le m \le M$ are known coeficients , $\alpha_n \ne 0, \phi_m \ne 0$, y N , M are positive integers and subject to the conditions

$$\frac{\partial^n u}{\partial t^n}(x,0) = g_n(x), \quad 0 \le n \le N-1, x \in \mathbb{R} \quad y \quad \frac{\partial^m u}{\partial x^m}(0,t) = f_m(t) \quad 0 \le m \le M-1, t \in \mathbb{R} \tag{12}$$

Define the following differential operators:

$$G_n = \frac{\partial^n}{\partial t^n}, \quad 0 \le n \le N \quad y \quad F_m = \frac{\partial^m}{\partial x^m} \quad 1 \le m \le M \tag{13}$$

with the convention $G_0 = F_0 = I$, is the identity operator. Then the equations (11) and (12)can be rewritten as

$$\sum_{n=0}^{N}\alpha_n(x,t)G_n u(x,t) = \sum_{m=1}^{M}\phi_m(x,t)F_m u(x,t) + f(x,t),(x,t)\in\Omega\subset\mathbb{R}^2 \quad (14)$$

$$G_n(x,0) = g_n(x), \qquad 0\le n\le N-1, x\in\mathbb{R} \quad y \quad F_m(0,t) = f_m(t) \qquad 0\le m\le M-1, t\in\mathbb{R} \quad (15)$$

where the inverse integral operator is formally defined as

$$G_N^{-1} = \int_0^{t_0=t}\int_0^{t_1}\ldots\int_0^{t_{N-1}} dt_N\ldots dt_1,\; F_N^{-1} = \int_0^{x_0=x}\int_0^{x_1}\ldots\int_0^{x_{M-1}} dx_M\ldots dx_1 \quad (16)$$

Applying (16)to (14) and (15) is obtained

$$u(x,t) = G_N^{-1}\left(\frac{f(x,t)}{\alpha_N(x,t)}\right) + \sum_{n=0}^{N-1}\left(\frac{t^n}{n!}\right)g_n(x) + \sum_{m=1}^{M}G_N^{-1}\left(\frac{\phi_m(x,t)}{\alpha_N(x,t)}F_m u(x,t)\right) - \sum_{n=0}^{N}G_N^{-1}\left(\frac{\alpha_n(x,t)}{\alpha_N(x,t)}G_n u(x,t)\right). \quad (17)$$

Using the ADM (1994), the following relationship is defined for equation (17)

$$u_0(x,t) = G_N^{-1}\left(\frac{f(x,t)}{\alpha_N(x,t)}\right) + \sum_{l=0}^{N-1}\left(\frac{t^l}{l!}\right)g_l(x), \quad (18)$$

and for $k\in\mathbb{N}_0$

$$u_{k+1}(x,t) = \left[\sum_{m=1}^{M}G_N^{-1}\left(\frac{\phi_m(x,t)}{\alpha_N(x,t)}F_m\right) - \sum_{n=0}^{N}G_N^{-1}\left(\frac{\alpha_n(x,t)}{\alpha_N(x,t)}G_n\right)\right]u_k(x,t) \quad (19)$$

in this way the solution can be represented by

$$u(x,t) = \sum_{k=0}^{\infty}u_k(x,t) \quad (20)$$

4.1 Approximate analytical solution for model of mean reversion

For the special case in which $N=1, M=2, \alpha_0 = r, \alpha_1 = -1, \phi_0 = 0, \varphi_1 = \beta[\theta - x], \phi_2 = \frac{\sigma^2}{2}x^2$, we obtain the terminal value problem consisting of the partial differential equation

$$\frac{\partial u}{\partial t} + \beta[\theta - x]\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\sigma^2 x^2 - ru = 0 \quad (21)$$

and the terminal condition

$$u(x,T) = g(x) = Max\{x-K,0\} \quad (22)$$

Applying the above formulas (18)and (19) the problem (21) and (22) is obtained

$$u_0(x,t) = g(x) \tag{23}$$

and for $k \in \mathbb{N}_0$,

$$\begin{aligned} u_{k+1}(x,t) &= \left[\sum_{m=1}^{2} G_1^{-1}\left(\frac{\phi_m(x,t)}{\alpha_1(x,t)} F_m \right) - \sum_{n=0}^{0} G_1^{-1}\left(\frac{\alpha_n(x,t)}{\alpha_1(x,t)} G_n \right) \right] u_k(x,t) \\ &= G_1^{-1}\left[-\beta[\theta - x]F_1 - \frac{\sigma^2}{2} x^2 F_2 + rG_0 \right] u_k(x,t) \\ &= \int_T^t \left[-\beta[\theta - x] \frac{\partial u_k(x,\tau)}{\partial x} - \frac{\sigma^2}{2} x^2 \frac{\partial^2 u_k(x,\tau)}{\partial x^2} + r u_k(x,\tau) \right] ds \end{aligned} \tag{24}$$

ie,

$$u_{k+1}(x,t) = \int_t^T \left[\frac{\sigma^2}{2} x^2 \frac{\partial^2 u_k(x,s)}{\partial x^2} - \beta x \frac{\partial u_k(x,s)}{\partial x} - r u_k(x,s) \right] ds + \int_t^T \beta\theta \frac{\partial u_k(x,s)}{\partial x} ds \tag{25}$$

By Theorem (2.1) in M. Bohner and Y. Zheng (2009), the recursively defined functions (23) and (25) can be represented explicitly as

$$u_k(x,t) = \left[\sum_{m=0}^{2k} \left\{ \sum_{v=0}^{m} \frac{(-1)^{m-v}}{v!(m-v)!} \rho_v^k \right\} x^m g^{(m)}(x) \right] \frac{(T-t)^k}{k!} + \beta\theta \sum_{m=0}^{2k} g^{(m)}(x) \frac{(T-t)^k}{k!} \tag{26}$$

in compact form,

$$u_k(x,t) = \left[\sum_{m=0}^{2k} \left\{ \sum_{v=0}^{m} \frac{(-1)^{m-v}}{v!(m-v)!} \rho_v^k \right\} x^m g^{(m)}(x) + \beta\theta \sum_{m=0}^{2k} g^{(m)}(x) \right] \frac{(T-t)^k}{k!},\ k \in \mathbb{N}_0 \tag{27}$$

Thus the solution is given by

$$u(x,t) = \sum_{k=0}^{\infty} \left\{ \left[\sum_{m=0}^{2k} \left\{ \sum_{v=0}^{m} \frac{(-1)^{m-v}}{v!(m-v)!} \rho_v^k \right\} x^m g^{(m)}(x) + \beta\theta \sum_{m=0}^{2k} g^{(m)}(x) \right] \frac{(T-t)^k}{k!} \right\},\ k \in \mathbb{N}_0 \tag{28}$$

where, $\rho_m = m(\frac{\sigma^2}{2}(m-1) + \beta)$ for all $m \in \mathbb{N}_0$.

Note that the boundary condition $g(x) = Max\{x - K, 0\}$ is a differentiable function and consequently the functions $g^{(m)}(x), 0 \le m \le 2k$ are not explicitly defined. To correct this deficiency, M. Bohner, F. Marin and S. Rodriguez (2013) propose an analytic function of the form

$$g(x) = \frac{1}{2}(x - K) + \frac{1}{2}\sqrt{(x-K)^2 + a^2\left(2\sqrt{2} - 1\right)} \tag{29}$$

which converges to the boundary condition, where K is the strike price and the parameter $a \in (0,1]$ is a nonnegative constant which serves as a control variable for convergence.

5. ALTERNATIVES NUMERICAL METHODS

This section provides a brief description of the Monte Carlo simulation method, binomial trees and Finite Differences that have been useful as alternatives numerical for option pricing under the objective probability $\mathbb{P}^*$ that will be compared with the Adomian decomposition method (ADM).

5.1 Monte Carlo simulation

To evaluate options using Monte Carlo simulation, we consider the following procedure:

1. M simulations are taken price dynamics for S_t under the objective probability $\mathbb{P}^*$ (equation (9)) as the expiration time of the option using the Euler numerical scheme.
2. Calculate the payoff of the derivative in each path
3. Calculate the average payoff of each trajectory to estimate the expected payoff in a risk neutral world and adjusted.
4. Excluding the expected payoff to the interest rate risk-free to obtain an estimate of the value of the derivative at time zero.

5.2 Binomial trees

Following the results of F. Marin (2010), assume that the life of an option on an asset that pays no dividends, with initial price x_0 and exercise price K, is divided into N subintervals of length Δt . Define $u_j^{(i)}$ as the value of the option at node (i, j), where the asset price at node (i, j) is given by

$$x_j^{(i)} = x_0 \prod_{k=1}^{j-1} u_k^{*(k-1)} \prod_{k=1}^{i-j+1} d_j^{(i-k)} \tag{30}$$

For the case of a European call option, the value on the date of maturity is given by $Max\left(x_j^{(i)} K, 0\right)$, Therefore

$$u_j^{(N)} = Ma'x\left(x_0 \prod_{k=1}^{j-1} u_k^{*(k-1)} \prod_{k=1}^{i-j+1} d_j^{(i-k)} - K, 0 \right), \qquad for \ j = 1,2,3,\cdots N \tag{31}$$

Also

$$u_j^{(i)} = e^{-r\Delta t}\left[p_j^{(i)} u_{j+1}^{*(i+1)} + \left(1 - p_j^{(i)}\right) d_j^{(i+1)} \right], \quad for \quad 0 \le i \le N-1 \quad and \quad 1 \le j \le i \tag{32}$$

Where dynamic transition probabilities under the objective probability $\mathbb{P}^*$ are given by

$$p_j^{(i)} = \frac{1}{2} + \frac{\left[\left(\frac{\theta\beta}{x_j^{(i)}} - \beta \right) - \frac{1}{2}\sigma^2 \right]\sqrt{\Delta t}}{2\sigma}, \quad u_j^{*(i)} = e^{\sigma\sqrt{\Delta t}}, \ d_j^{(i)} = \frac{1}{u_j^{*(i)}}. \tag{33}$$

5.3 Finite differences method

As the numerical dominance of the equation (10) is unbounded and for numerical approximation is important to have a bounded domain such that it is possible to calculate the solution. The limited number domain can be chosen according to different criteria, see R. Kangro et. al (2000) for an example.

Denote by $[0;b]$ the domain of the variable x, where b is chosen so that the interval includes the exercise price and the initial price and denoted by $[0,T]$ the domain of the variable t.

Then we define the numerical domain as:

$$(x,t) \in [0;b] \times [0,T]$$

with the nodes

$$x_i = ih_1; 0 \le i \le N_x$$
$$t_j = nk; 0 \le j \le N_t$$

$$N_x h_1 = b; \ N_t k = T$$

The numerical aproximation of exact solution $u(x_i, t_j)$ is denoted by U_{ij}.

The partial differential equation (10) will be transformed into a set of difference equations which are solved iteratively.

Then the approximations for the partial derivatives are given by

$$\begin{aligned}
\frac{\partial u}{\partial x}(x_j, t_i) &= \frac{U_{i+1,j+1} - U_{i+1,j-1}}{2h} + O(h) \\
\frac{\partial u}{\partial t}(x_j, t_i) &= \frac{U_{i+1,j} - U_{i,j}}{k} + O(k) \\
\frac{\partial^2 u}{\partial x^2}(x_j, t_i) &= \frac{U_{i+1,j+1} + U_{i+1,j-1} - 2U_{i+1,j}}{h^2} + O(h^2)
\end{aligned} \tag{34}$$

Replacing the partial derivatives of equation (10) by the approaches given in (34) yields the numerical scheme

$$U_{i,j} = a_j U_{i+1,j-1} + b_j U_{i+1,j} + c_j U_{i+1,j+1} \tag{35}$$

where,

$$\begin{aligned}
a_j &= \frac{k}{1+rk}\left(\frac{1}{2}\sigma^2 j^2 - \lambda_j\right) \\
b_j &= \frac{k}{1+rk}\left(\frac{1}{k} - \sigma^2 j^2\right) \\
c_j &= \frac{k}{1+rk}\left(\frac{1}{2}\sigma^2 j^2 + \lambda_j\right) \\
\lambda_j &= \frac{\beta(\theta - jh)}{2h}
\end{aligned}$$

With boundary conditions

$$\begin{aligned}
U_{i+10} &= U_{i0} = \ldots.. = U_{00} = 0 \\
U_{i+1N_x} &= U_{iN_x} = \ldots. = U_{0N_x} = b - K \\
U_{N_t\, j+1} &= U_{N_t\, j} = \ldots. = U_{N_t 0} = Max\{jh - K, 0\}
\end{aligned}$$

Note that for $0 < \theta \le b$ and $h > 0$, if $k \le \left(\frac{\sigma h}{b}\right)^2$, the coeficients a_j, b_j and c_j are no negatives for $0 \le j \le N_x$.

It is easy to check that these schemes also fulfill the conditions of positivity and monotonicity described in F. Marin and M. Bastidas (2012). A strategy for the analysis of stability, consistency and convergence can be found in Smith (1985) combined with the equivalence theorem of Lax-Richtmyer.

6. NUMERICAL RESULTS

In this section we present some numerical experimental results for the valuation of a European call option under the objective probability $\mathbb{P}^*$ using the Adomian decomposition method with different expiration dates, compared with the results obtained using Monte Carlo simulation, binomial trees and finite differences.

Table 1 shows the results of a European call option valuation under the objective probability $\mathbb{P}^*$ with different expiration dates. $K = 10$, $S_0 = 19$, $\theta = 10.1$, $\sigma = 0.3$, $r = 0.05$, $\beta = 0.0023$.

Time	Adomian	Monte Carlo	Binomial trees	Finite differences
3 months	9.005748	9.014229	8.883107	8.953930
2 months	9.003832	9.071600	8.921899	8.963053
1month	9.001916	9.055988	8.960864	8.975638

Table 1: Comparison of the results of a European call option valuation under the objective probability $\mathbb{P}^*$ with various expiry times calculates using binomial trees with 100 time steps, Monte Carlo simulation with 1000 samples (unused antithetic variable variance reduction) and Finite differences with $h = 0.25$.

Figure 1 shows a empirical sensitivity analisys of the parameter a to the payment function $g(x) = \frac{1}{2}(x-K) + \frac{1}{2}\sqrt{(x-K)^2 + a^2(2\sqrt{2}-1)}$ regard to the acurrate ate of the option price for $T = 1/4$. It is noted that when the control parameter a approaches zero, the option price under the ADM scheme, approaches of the option price.

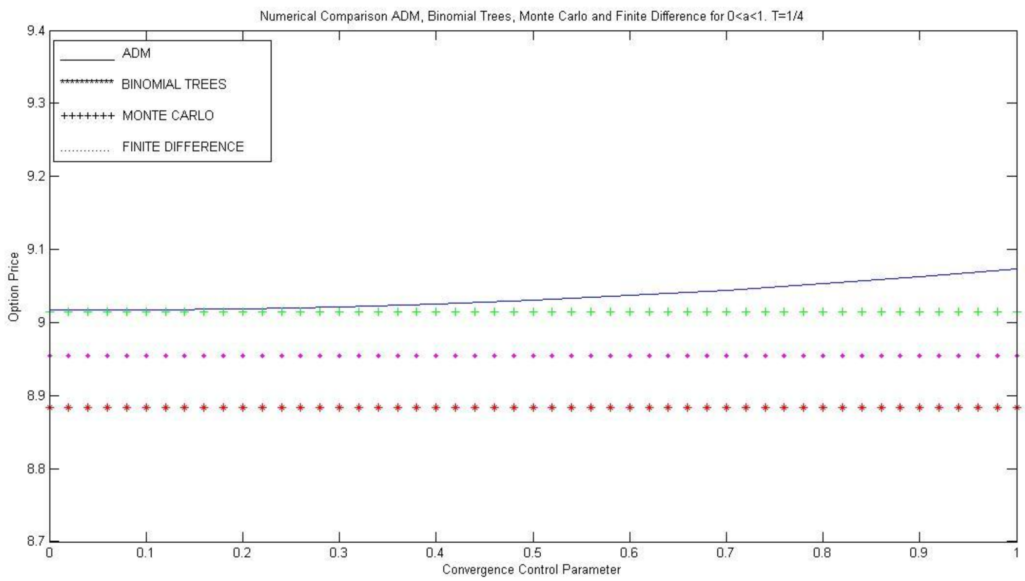

Figure 1: Numerical comparison ADM, Binomial trees, Monte Carlo and finite differences for $0 < a \leq 1$. (3 Months)

Figures 2, 3 and 4 shows the value of the European call option obtained by the Adomian decomposition method under the objective probability $\mathbb{P}^*$, compared to Monte Carlo simulation methods, binomial trees, and finite differences for expiration dates $T = 1/2$, $T = 1/6$ and $T = 1/4$ respectively.

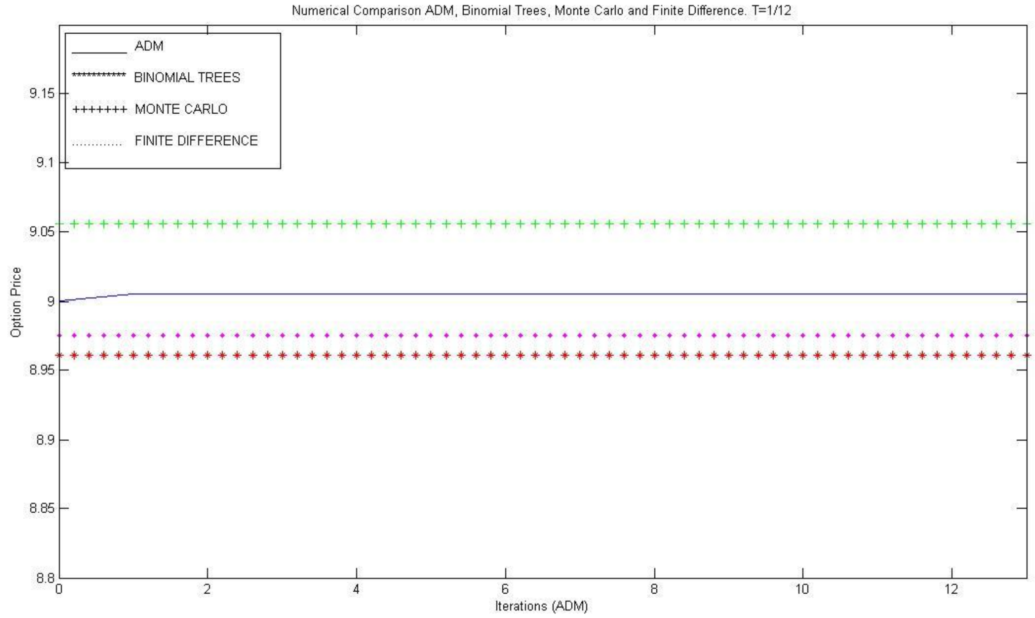

Figure 2: Numerical comparison ADM, Binomial trees, Monte Carlo and finite differences. (1 Month)

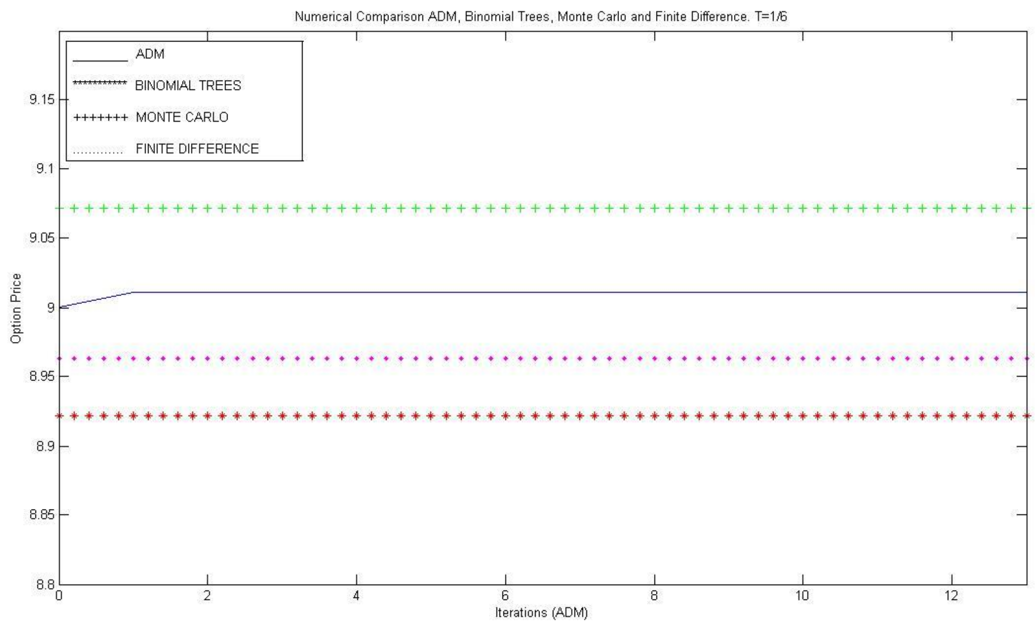

Figure 3: Numerical comparison ADM, Binomial trees, Monte Carlo and finite differences. (2 Months)

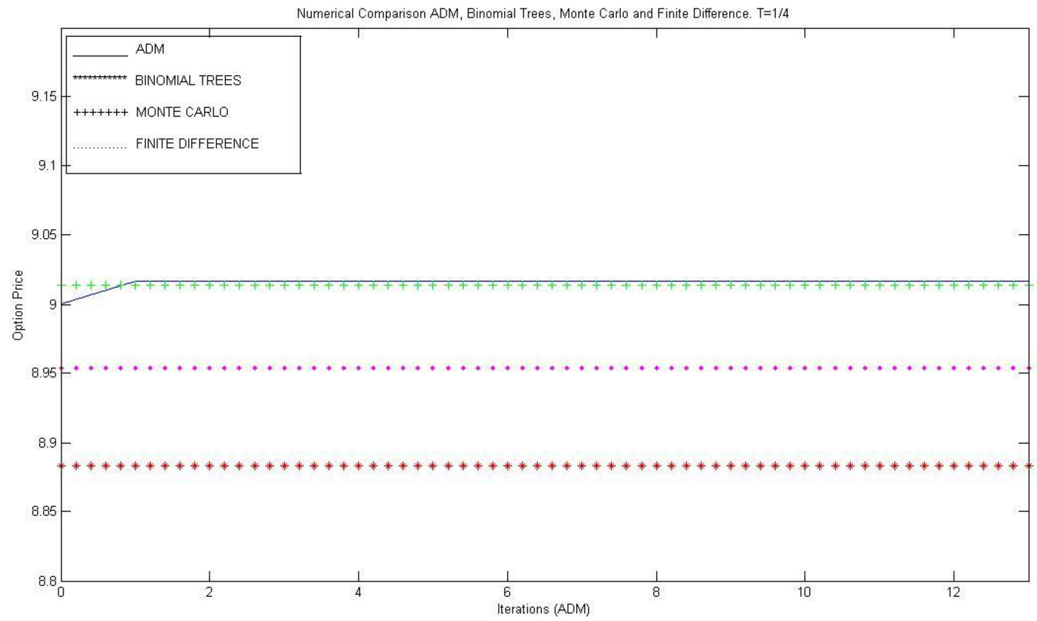

Figure 4: Numerical comparison ADM, Binomial trees, Monte Carlo and finite differences. (3 Months)

7. CONCLUSIONS

In this paper we obtain an analytical approach to the partial differential equation (10) whose solution represents the value of a European call option on processes mean reversion with proporsional noise under objective probability measure $\mathbb{P}^*$.

The implementation of this analytical approach is simple and provides appropriate solutions for payment general analytic functions, and consistent with the boundary condition of the Black Scholes model. Because of its rapid convergence, the computational cost can be reduced if you take a few iterations ($k = 14$ works well). To empirically test the convergence of the approximation, was used the control variable $a \in (0,1]$ for different values. Some numerical experiments were designed to graphically illustrate the rapid converegencia's (ADM) which was compared with other alternative methods to value options. The results obtained with (ADM) are very close to those obtained with the other methods, these results suggest that (ADM) is a powerful method that can be used in evaluating other options. For example (ADM) can be used to value European put options considering an overall payoff function, similar to the function $g(x)$ in equation (29).

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