NUMERICAL MODELING ON ELECTRIC RESPONSE OF FIBRE-ORIENTATION OF COMPOSITES WITH PIEZOELECTRICITY

Cheuk-Yu Lee, Qing H. Qin and Guy Walpole
Research School of Engineering, Australian National University, Canberra, ACT 2601, Australia

ABSTRACT

A three point bending test is considered for analysing the effect of fibre orientation on electric response of fibre reinforced composites attached a piezoelectric sensor. The study involves finite element simulation on a bent unidirectional carbon fibre reinforced plastic-piezoceramic composite laminate. Specifically, finite element analysis is used in order to establish a relationship between the fibre angle of the carbon-fibre and the electric response of the piezoceramic, under 3 point bending. The validity of the FE results is compared experimental observations.

Keywords: Fibre reinforce composite; Piezoelectric; Finite element; Three point bending test

1. INTRODUCTION

In materials science, composite laminates are usually assemblies of layers of fibrous composite materials which can be joined together to provide required engineering properties, such as specified in-plane stiffness, bending stiffness, strength, and coefficient of thermal expansion [1,2]. On one hand, it is noted that fibre orientation is an important aspect of fibre-reinforced composites due to the dependence of its properties on fibre orientations significantly. On the other hand, Adherence of a piezoelectric material to the surface of a carbon fibre-reinforced polymer (CFRP) allows for the measurement of strain within the CFRP as a function of charge (see Fig. 1). Under a bending strain, the measured charge could be used directly as a feedback signal to show the health situation of the structure.

It should be mentioned that the strong demand for high performance structures has driven a new development of “smart materials and structures” in recent years. Piezoelectric smart structures composed of passive elastic materials and active piezoelectric materials have been recently developed. They are now gaining acceptance in a variety of engineering applications, such as adaptive shape, self-diagnosis, transducers for underwater and biomedical imaging, active damping, robotic manipulators, high precision devices, micro sensors/actuators, vibration and noise control, etc. By employing piezoelectric materials, it is feasible to achieve an accurate response to monitor and to provide effective control of flexible structures. It is expected that research on intelligent structures with piezoelectric elements will receive increasing attention for many years to come. During the past decades, piezoelectric composite materials have been developed by combining piezoceramics with passive non-piezoelectric polymers. Superior properties are achieved by taking advantage of most profitable properties of each constituents and a great variety of structures have been produced. To enhance understanding of the electromechanical coupling mechanism in piezoelectric materials and to explore their potential applications in practical engineering, numerous investigations,
either analytically or numerically [3-10], have been conducted over the past decades. Banno [11] studied discontinuous reinforcement through a cubes approach. In a more rigorous attempt to account for the interactions that exist among continuous fibres at finite concentrations, Grekov et al. [12] developed a concentric cylinder model to predict the effect electroelastic moduli of a continuous fiber reinforced composite. Dunn and Taya [13] estimated the effective properties using dilute, self-consistent, Mori-Tanaka and differential micromechanical models. Chen [14] obtained some formulae for estimates of the overall thermoelectroelastic moduli of multiphase fibrous with self-consistent and Mori-Tanaka schemes. The derivation are based on an overall constraint tensor. Benveniste and Dvorak [15] showed that in two-phase fibrous piezocomposites uniform strain and electric fields can be generated by certain loading conditions. The concept of uniform fields was further elaborated by Benveniste [16,17] in two-, three- and four-phase composites with cylindrical microstructures. Later, Qin et al [18] presented a set of formulae to estimate the damage constants of thermo-piezoelectric ceramics due to many microcracks. Qin and Mai [19] also presented a Green function for biomaterial piezoelectricity. More recently, Qin [20] integrated boundary element method and micromechanics models for determining effective properties of piezoelectric materials.

In this paper, the 3 point bending of a unidirectional CFRP-Piezoceramic laminate was simulated using FEA, with the charge across the piezoelectric being conjectured to be a function of the fibre angle of the CFRP. ABAQUS is used as it features special purpose piezoelectric elements capable of simulating the electro-mechanical coupling of such materials. It is conjectured that the relationship observed between the electric response of the piezoceramic and the fibre angle of the CFRP is linear. Verify of this conjecture could be provided using a testing method which applied bending to the samples without allowing significant twisting of the samples, an effect which is prevalent within the existing experimental method, where the samples are simply supported.

2. CONSTITUTE BEHAVIOUR

From an engineering perspective, the behaviour of piezoelectric materials is governed by the principles of both mechanical equilibrium and electric flux equilibrium. Assuming small displacements and electric fields, there is an implied linear relationship. Subsequently, the linear constitutive behaviour that defines the materials coupling can be represented using one of the four forms shown below. The first form shown is known as stress-charge relationship, which is derived using the elastic enthalpy function [6]

\[ H(\varepsilon_{ii}, E_k) = \frac{1}{2} D_{ijkl} \varepsilon_{ij} \varepsilon_{kl} - \varepsilon_{ik} E_j - \frac{1}{2} \kappa_{ii} E_k E_j \]  \hspace{1cm} (1)

With the linear constitutive equations given by:

\[ \sigma_{ij} = \frac{\partial H}{\partial \varepsilon_{ij}} = D_{ijkl} \varepsilon_{kl} - \varepsilon_{ik} E_j \]
\[ D_{ij} = -\frac{\partial H}{\partial E_i} = \varepsilon_{ij} \varepsilon_{kl} + \kappa_{ij} E_i \]  \hspace{1cm} (2)

where \(\sigma_{ij}\) and \(\varepsilon_{ij}\) are the stress and strain tensors respectively, and \(D_{ijkl}\) is the elastic stiffness. \(E_i\) is the electric field, and \(D_{ij}\) the electric displacement vector. \(\kappa_{ij}\) is the dielectric constant.

3. PIEZOCERAMIC-CFRP LAMINATE BENDING

3.1. Three point bending test

The three point bending test is used to investigate the effect of fibre-orientation of CFRP on electric response of the composite. The benefits of the three point test are the ability to easily prepare samples and that the point of failure is predictable. The setup involves two simple supports at the base of sample; with an applied load \(F\) located in the middle of the beam is shown in Figure 1.

In our work, samples of a new type of piezoelectric composite have been prepared using a laminate of a piezoceramic with unidirectional CFRP (see Figure 2). The specific dimensions of the sample are illustrated in Figure 2. The properties of both materials are listed below in Table 1. The CFRP is a standard graphite fibre and epoxy matrix with 50 vol% fibre. The piezoceramic is a commercially sourced with properties taken from the relevant PSI-5A4E data sheet (Piezo Systems, 2007). The experimental setup for the physical testing of the laminate utilizes the applied displacement \(\delta\), of 500μm. An electrometer is used to measure the charge across the piezoceramic, via electrodes which are attached to be top and bottom faces. The electro-mechanical coupling of the laminate is analysed using physical testing in the form of a 3 point bending test [21]. The aim is to establish a relationship between the electric response of the piezoelectric ceramic and the fibre orientation of the CFRP.

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Figure 2 Dimensioned views of CFRP-Piezoceramic samples. All measurements are in millimetres (mm).

Table 1: Material properties used within the Piezoceramic-CFRP laminate FE models

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>66 GPa</td>
</tr>
<tr>
<td>$E_2$</td>
<td>52 GPa</td>
</tr>
<tr>
<td>$E_3$</td>
<td>52 GPa</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
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</tr>
<tr>
<td>$\nu_{13}$</td>
<td>0.25</td>
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<tr>
<td>$\nu_{23}$</td>
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<tr>
<td>$G_{12}$</td>
<td>26.4 GPa</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>23.4 GPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>23.4 GPa</td>
</tr>
<tr>
<td>$d_{33}$</td>
<td>$390 \times 10^{-12}$ mV$^{-1}$</td>
</tr>
<tr>
<td>$d_{13}$</td>
<td>$-190 \times 10^{-12}$ mV$^{-1}$</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>$1.594 \times 10^{-8}$ NV$^{-1}$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>135 GPa</td>
</tr>
<tr>
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<td>7 GPa</td>
</tr>
<tr>
<td>$E_3$</td>
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<tr>
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</tr>
<tr>
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<td>0.25</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.025</td>
</tr>
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<td>12.3 GPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>3.5 GPa</td>
</tr>
</tbody>
</table>

3.2. Theoretical Electro-Mechanical Response

There are various bending configurations that can be implemented which utilize the electromechanical coupling of the piezo-ceramic in different ways. Piezoelectric transducers of this form have two types of application: actuators and sensors. Actuators convert electrical energy to mechanical energy and are often referred to as motors. Sensors convert mechanical energy into electrical energy and are often referred to as generators. The physical testing of the
lamine samples is being conducted in a sensor configuration, where an applied deflection is the control variable and the resulting electric response of the piezo-ceramic is being observed. It is anticipated that the observed response of the piezoceramic in both the case of physical testing and FE modelling will be consistent with a theory presented by Dunsch & Breguet [22]. Firstly, the geometry of the deformed samples feature an average radius of curvature as illustrated in Figure 1, where the control variable $\delta$ is expressed according the following relationship:

$$\delta = \rho[1 - \cos\left(\frac{l}{2\rho}\right)]$$

(3)

where the radius of curvature can be related to the change in curvature according to Euler beam theory:

$$\frac{1}{\Delta \rho} \sum_i E_i I_i = \frac{\Delta M_a}{E_i I_i + E_{piezo} I_{piezo} + E_{CFRP} I_{CFRP}}$$

(4)

with $\Delta M_a$ being the change in bending moment resulting from the applied loading at the centre.

The elastic response of the sample is governed by the fundamental assumption of static force balance between the layers. Which, where applied utilizing the Hellinger-Reissner principle [6] is equivalent to:

$$\int \delta \varepsilon \sigma dV - \int \delta u^T F_y dV - \int A \delta u^T F_z dS = 0$$

(5)

where $\delta \varepsilon$ and $\delta u$ are the virtual strain and displacement vectors respectively, and $F_y$ and $F_z$ are the body force and surface force vectors respectively. The charge $Q$ that is measured between the electrodes of the physical sample can be observed within the FE model as a function of electric displacement $D$, according to Gauss’ Law:

$$Q = \int_{A_e} D_z dA_e$$

(6)

where $A_e$ is the area of the piezoceramic cell. The electric displacement in the third direction being found using the fundamental constitutive equations for piezoelectricity illustrated in equation, which given the geometry of the test samples, is expressed as:

$$\varepsilon_z = S_{13} \sigma_1 + d_{31} E_z$$

$$D_3 = d_{31} \sigma_1 + \kappa_{33} E_3$$

(7)

4. PIEZOCERMIC-CFRP LAMINATE FE MODELS
The physical testing conducted [21] is concerned with unidirectional CFRP laminated at angles of 0°, 45°, and 90° degrees with respect to the longitudinal direction between the two supports of the 3 point bending test. This means that when modelling the laminate with fibre angles of 0°, and 90°, there are two planes of symmetry as illustrated to be exploited. In the case of the CFRP with a fibre angle of 45°, there is a single plane of symmetry. These planes of symmetry are demonstrated in Figure 3.

Figure 3 The laminate samples feature 2 planes of symmetry, where the modelled region is highlighted, viewed from the underside. Note that the 45o sample does not feature the longitudinal plane of symmetry.

A full 3D modelling approach based on ABAQUS software has been used when modelling the laminate. This is in contrast to more conventional FE modelling approaches of modelling a composite layup which are unsuitable for this application. Often, specific 2D composite layup elements are used on the assumption of a very thin laminate also the aspect ratio of the samples does not warrant this approach. Alternately, 2D plane strain conditions are implemented when the sample has a large width and constant cross section. Again, this approach cannot be utilised due to the narrowness of the sample and various in cross-section around the piezoceramic. Subsequently, a 3D modelling approach was selected as this was the only method which could ensure an accurate model.
The applied force on the samples was applied using a linear ramp until the prescribed displacement $\delta$ was observed at 0.5mm. Figure A1 in Appendix A at the end of this paper illustrates the mesh density used on the models where all elements are linear displacement quadrilaterals with aspect ratio 1. The discontinuous change in mesh density across the piezoceramic-CFRP interface is accounted for using a tie constraint, which also simulates a perfect bond at this interface.

5. RESULTS FOR THE PIEZOCERMIC-CFRP LAMINATE SAMPLE

The stress distribution in the longitudinal direction (between the supports) through each of the 3 sample configurations is illustrated below in Figures 4, 6 and 8, demonstrating the mechanical response of the samples. Figures 5, 7 and 9 show the electric response of the sample configurations by illustrating the electric displacement observed on the bottom surface of the piezoceramic. This surface electric displacement was observed to be the inverse of that on the top face of the piezoceramic, indicating an electric potential across the piezoceramic cell. As such, the ABAQUS FE results can be related to the electric charge observed in the physical testing [21] using Gauss’ Law. The electro-mechanical response within the piezoceramic for each fibre is listed in Table 2. Note that whilst the stress distributions are only shown for the ABAQUS FE modelled region, the electric displacement contour plots have been constructed for the entire bottom face of the piezoceramic cell. The electric displacement on the top face of the piezoceramic is assumed to be the inverse of the bottom face due to the similar displacement profiles relative to the neutral plane, where the poling direction is between two faces. This means that the amount of positive charge which builds up on the bottom surface will be equivalent to the amount of negative charge which accumulates on the top face.

Figure 4 Side and bottom views of the stress distribution in the longitudinal direction through the 0° sample.

Figure 5 Electric displacement distribution on the bottom face of the piezoceramic for 0° sample.
Figure 6 Side and bottom views of the stress distribution in the longitudinal direction through the 45° sample.

Figure 7 Electrical displacement distribution on the bottom face of the piezoceramic for 45° sample.

Figure 8 The stress distribution in the longitudinal direction through the 90° sample.
Figure 9 Electrical displacement distribution on the bottom face of the piezoceramic for 90° sample.

Table 2 Summary of the electro-mechanical response of samples according to fibre angle.

<table>
<thead>
<tr>
<th>Fibre Angle</th>
<th>Peak Displacement</th>
<th>Applied Load</th>
<th>Peak Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>500μm</td>
<td>62.5N</td>
<td>7.3E-8C</td>
</tr>
<tr>
<td>45°</td>
<td>500μm</td>
<td>10.1N</td>
<td>5.0E-8C</td>
</tr>
<tr>
<td>90°</td>
<td>500μm</td>
<td>8.6N</td>
<td>4.2E-8C</td>
</tr>
</tbody>
</table>

The FE results presented in this work have been produced primarily as a means of validated the results presented in [21]. As such, it is worth presenting the FE results in direct comparison to those gained from physical testing [21], as shown in Figure 10.

Figure 10 Charge observed across the piezoceramic, as a function of applied displacement. Note the overlayed physical testing results [21].

6. DISCUSSION ON RESULTS
Referring first to Figure 10, note the correlation between FE simulation results and those from the physical testing [21]. Furthermore, note that the charge observed through both means shows a discernable response according to fibre orientation. The relationship between the physical testing and FE simulation is shown to have peak error of approximately 10%, observed within the intermediate displacement range. The observed discrepancy between the two analyses methods can be explained fundamentally as a function of two key factors. Firstly, ABAQUS analyses piezoelectric materials using the linear constituent relationships defined in Eq (2) or Eq. (7). This assumption of linearity between applied mechanical loading and resulting electric displacement, whilst accurate for very small displacements, becomes less accurate given larger displacements. It is likely that the 500μm applied displacement used during physical testing of the samples has exceeded the linear range of the piezoceramics response. The electro-mechanical response in this non-linear range is not be accounted for in our analysis within
ABAQUS and subsequently the FE simulations maintain a linear response throughout, a relationship to which the physical testing alludes strongly in the lower displacement range, without replicating exactly at peak displacements. Furthermore, a second factor accounting for the discrepancy between the two testing methods is the potential for fibre debonding at the piezoceramic-CFRP interface, particular at peak displacements. If debonding occurs during physical testing, the effective displacement at the ends of the piezoceramic will be reduced with respect to the neutral axis, resulting in a lower charge across the cell, than if the piezoceramic had maintained a perfect bond with the CFRP. The potential for debonding to occur is highlighted in Figures A2-A4 (Appendix A), where singularities can be observed at the ends of the piezoceramic-CFRP interface. It was obvious that a stress singularity at such an interface is indicative of the formation of a crack tip, the FE results implies that debonding is to be expected, an issue which is not observed in the physical testing results. This causes a discrepancy between the FE results, where a perfectly bonded piezoceramic-CFRP interface was implemented without a cohesive zone model. Subsequently, the onset of interface debonding is not accounted for within the FE results.

Whilst the results presented within this work serve primarily as a means of verifying the physical testing results given in [21], the FE results are useful for providing further insights into response of the sample, which cannot be obtained through physical testing alone. Specifically, whilst a clear relationship between the electric response of the piezoceramic cell and the fibre orientation of the CFRP was observed using both testing methods, the FE model provides the best insight into the cause of this relationship. Considering Euler-Bernoulli beam theory, it has been conjectured that the cause of the relationship is the varying radius of curvature (RoC) of the samples, a function of the longitudinal elastic modulus $E$ of the CFRP. Consequently, whilst the deflection of the piezoceramic is constant at the centre, the deflection through the bulk piezoceramic varies slightly according to the modulus $E$ of the CFRP perpendicular to the applied loading, the value of which varies considerably according to the fibre angle given its anisotropic nature. Observing the RoC of the piezoceramic is inherently difficult within a physical testing environment, especially given how sensitive the piezoceramic is to small changes in deflection. Subsequently, accurate observation of the variation of RoC through the samples on an adequately small scale is best achieved through the analysis of the FE Models. Subsequently, Figure 11 shows the exact deflection of the samples all the centreline of the piezoceramic cell for each CFRP fibre angle.

![Figure 11 Deflection of the piezoceramic from the neutral axis according to fibre angle.](image)

The different displacement profiles illustrated in Figure 11 indicate that each fibre angle produces a unique stress distribution through the piezoceramic cell. Given the inherent electro-mechanical coupling within the cell, this leads to the varied electric response which has been observed. Figure 10 and Table 2 both allude to the relationship between fibre orientation and electric charge being non-linear, with the charge differential between the 0° and 45° samples being far smaller than between 45° and 90°. This effect is the result of a twisting which occurs during the bending of the 45° sample given the simply-supported boundary conditions of both the FE simulation and the physical testing reported in [21], this effect is visible in both the stress distribution of Figure 6, and the electric displacement of Figure 7. If another testing method were used featuring more suitable boundary conditions, it is anticipated that this relation could be shown to be linear.

7. SUMMARY

In this work, a FE simulation of 3 point bending of a unidirectional CFRP-piezoceramic laminate was undertaken, correlated with the equivalent physical testing results reported in [21]. Based on the FEA results, a number of
conclusions can be made: (1) A relationship between the fibre angle of the unidirectional CFRP and the charge across the piezoceramic was established, with this result being verified according to the correlation with the physical testing reported in [1]; (2) The cause of this relationship was verified as anisotropic nature of the CFRP causing different displacement profiles through the piezoceramic as a function of fibre angle, which is consistent with Euler-Bernoulli beam theory. Furthermore, it is conjectured that using an appropriate testing method, this relationship could be demonstrated as linear.

Appendix A: Finite element meshes used in ABAQUS software

Figure A1 Mesh densities used in each FE model. Note aspect ratio of 1 is maintained for all elements. The discontinuous mesh densities across the piezoceramic-CFRP interface is accommodated by using a tie constraint. The Piezoceramic is modelled using 20,416; 72μm 3D piezoelectric elements. The CFRP is modelled using 53,647; 170μm 3D piezoelectric elements

Figure A2 Close-up view of stress singularity at the ends of the piezoceramic within the 0° sample FE model.

Figure A3 Close-up view of stress singularity at the ends of the piezoceramic within the 45° sample FE model.

Figure A4 Close-up view of stress singularity at the ends of the piezoceramic within the 90° sample FE model.
8. REFERENCES