RELIABILITY FORECAST FOR A SOLAR PHOTOVOLTAIC (PV) SYSTEM INTEGRATED WITH GREENHOUSE BY EMPLOYING ALGEBRA OF LOGICS

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ABSTRACT

In this paper, the author has been considered a solar photovoltaic (PV) system integrated with greenhouse system to analyze its stochastic behavior and estimate the reliability by Boolean function technique. The authors have used algebra of logics and Boolean function technique for the purpose of formulation of mathematical model and its solution. Reliability and M.T.T.F. for the system have evaluated. Some particular cases have also given to improve practical utility of the model. Graphical illustration followed by a numerical example has appended in last to highlight the important results of study. It is observed that reliability of the system decreases rapidly in case, when failures follow Weibull time distribution, while it decreases smoothly and in better way when failures follow exponential time distribution.

Keywords: Boolean function technique, Reliability, Mean time to failure, Special cases.

1. INTRODUCTION

In this study, the performance of the solar photovoltaic (PV) system integrated with greenhouse has been discussed. The power produced by the PV system is used to operate the required heating/cooling equipments inside the greenhouse. A greenhouse may be defined as a sophisticated structure, providing ideal conditions for satisfactory plant growth and production throughout the year. To maintain favorable conditions in the greenhouse during off/pre and past harvesting some additional sources are required. The solar PV system is one of the energy sources which works at the lowest cost. The block diagram of solar PV system has been shown in fig-1. The various components of the system are as follows:

- Solar Panel
- Logic based charge controller
- Battery bank
- Converter DC/AC

1.1 Solar Panel

A 1.2 KWP photovoltaic system has been integrated to greenhouse for operating all equipments for heating and cooling. Each module has an effective area of 0.72 M² and produces 75-peak watt power (at 1000 w/m² solar irradiance and 25°C cell temperature). There are 16 modules in the complete system to get the required amount of voltage and current.

1.2 Logic based intelligent solar charge controller

The logic based solar charge controller controls the charging of battery. It ensures that when the battery gets completely charged, the charging current to the battery bank is stopped and similarly when the battery voltage falls below a threshold value, the charging of the battery is restarted. This is important to prevent over charging the battery bank and also to ensure that when battery voltage falls below a predefined value, they are recharged. This is important to ensure long battery life.

1.3 Battery Bank

The battery bank is the principal energy storage device to ensure continuous supply even when the solar panel is unable to produce any power (under low light intensity conditions or during night time). This battery bank may have up to 12 batteries, each rated at 6V, 180Ah. The batteries used are lead acid type and are deep cycle batteries, i.e. they can discharged more of there stored energy while still maintaining long life.

1.4 DC/AC Grid interactive sine-wave inverter

To drive the AC loads a DC/AC converter rated at 3.0KVA and providing 220V/50Hz AC is used.
In this model, the author has been used algebra of logics to evaluate reliability and mean time to failure of this solar PV system. The reliability parameters of this system have obtained in two different cases:
When failures follow weibull time distribution.
When failures follow exponential time distribution.
A numerical example with its graphical illustration has also been appended at the end to highlight important results of the study.

2. ASSUMPTIONS
The associated assumptions are as follows:

- Initially, all the equipments are good and operable.
- The state of each component and of the whole system is either good or fail.
- The states of all components of the system are statistically independent.
- The failure times of all components are arbitrary.
- Supply between any two components of the system is hundred percent reliable.
- There is no repair facility.
- The reliability of each component is known in advance.
- Battery bank consists of six batteries in parallel redundancy.
- There are two standby redundant DC/AC converters and on failure of one, second can be online through a perfect switching device.

3. NOTATIONS USED
The list of notations is as follows:

- \(x_1\): State of solar panel.
- \(x_2\): State of logic based charge controller.
- \(x_i (i = 3, 4, \ldots, 8)\): States of batteries in battery bank.
- \(x_y, x_{11}\): States of DC/AC converter.
- \(x_{10}\): State of switching device.
- \(C_i (i = 1, \ldots, 4)\): Cables to connect two components.
- \(x_i, \forall i\): is 1 for good, is 0 for bad.
- \(x_i'\): Negation of \(x_i\).
- \(\land / \lor\): Conjunction / Disjunction.
- \(R_S\): Reliability of the system as a whole.
- \(R_i\): Reliability of the component corresponding to system state \(x_i\).
- \(R_{SW}(t) / R_{SE}(t)\): Reliability functions when failure follows Weibull/exponential time distribution.
- \(M.T.T.F.\): Mean time to system failure.

4. FORMULATION OF MATHEMATICAL MODEL
The conditions of capability of successful operation of considered solar PV system, integrated with greenhouse, in terms of logical matrix are expressed as follows:
5. SOLUTION OF THE MODEL

By the application of algebra of logics, equation (1) may be written as

\[
F(x_1, x_2, \ldots, x_{11}) = [x_1 \quad x_2 \quad \cdots \quad x_{11}] 
\]

\[
\frac{\bigwedge_{i=1}^{11} \left( x_i \right)}{x_1 \quad x_2 \quad \cdots \quad x_{11}} = \cdots (1)
\]

\[
\frac{\bigwedge_{i=1}^{11} \left( x_i \right)}{x_1 \quad x_2 \quad \cdots \quad x_{11}} = \cdots (2)
\]

Fig-1: Block diagram of solar PV system
\[
\begin{bmatrix}
    x_3 & x_9 \\
    x_4 & x_9 \\
    x_5 & x_9 \\
    x_6 & x_9 \\
    x_7 & x_9 \\
    x_8 & x_9 \\
    x_3 & x_{10} & x_{11} \\
    x_4 & x_{10} & x_{11} \\
    x_5 & x_{10} & x_{11} \\
    x_6 & x_{10} & x_{11} \\
    x_7 & x_{10} & x_{11} \\
    -x_8 & x_{10} & x_{11}
\end{bmatrix}
\]

Now substituting the following in equation (3):

\[
A_1 = \begin{bmatrix} x_3 & x_9 \end{bmatrix} \quad \ldots(4)
\]

\[
A_2 = \begin{bmatrix} x_4 & x_9 \end{bmatrix} \quad \ldots(5)
\]

\[
A_3 = \begin{bmatrix} x_5 & x_9 \end{bmatrix} \quad \ldots(6)
\]

\[
A_4 = \begin{bmatrix} x_6 & x_9 \end{bmatrix} \quad \ldots(7)
\]

\[
A_5 = \begin{bmatrix} x_7 & x_9 \end{bmatrix} \quad \ldots(8)
\]

\[
A_6 = \begin{bmatrix} x_8 & x_9 \end{bmatrix} \quad \ldots(9)
\]

\[
A_7 = \begin{bmatrix} x_3 & x_{10} & x_{11} \end{bmatrix} \quad \ldots(10)
\]

\[
A_8 = \begin{bmatrix} x_4 & x_{10} & x_{11} \end{bmatrix} \quad \ldots(11)
\]

\[
A_9 = \begin{bmatrix} x_5 & x_{10} & x_{11} \end{bmatrix} \quad \ldots(12)
\]

\[
A_{10} = \begin{bmatrix} x_6 & x_{10} & x_{11} \end{bmatrix} \quad \ldots(13)
\]

\[
A_{11} = \begin{bmatrix} x_7 & x_{10} & x_{11} \end{bmatrix} \quad \ldots(14)
\]

\[
A_{12} = \begin{bmatrix} x_8 & x_{10} & x_{11} \end{bmatrix} \quad \ldots(15)
\]
We obtain

\[
f(x_3, x_4 = -, x_{11}) = \begin{bmatrix}
A_1 \\
A_2 \\
A_3 \\
A_4 \\
A_5 \\
A_6 \\
A_7 \\
A_8 \\
A_9 \\
A_{10} \\
A_{11} \\
A_{12}
\end{bmatrix}
\]

Using orthogonalisation algorithm, equation (16) may be written as:

\[
f(x_3, x_4 = -, x_{11}) = \begin{bmatrix}
A_1 \\
A'_1 & A_2 \\
A'_1 & A'_2 & A_3 \\
A'_1 & A'_2 & A'_3 & A_4 \\
A'_1 & A'_2 & A'_3 & A'_4 & A_5 \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A_6 \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A'_6 & A_7 \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A'_6 & A'_7 & A_8 \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A'_6 & A'_7 & A'_8 & A_9 \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A'_6 & A'_7 & A'_8 & A'_9 & A_{10} \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A'_6 & A'_7 & A'_8 & A'_9 & A'_{10} & A_{11} \\
A'_1 & A'_2 & A'_3 & A'_4 & A'_5 & A'_6 & A'_7 & A'_8 & A'_9 & A'_{10} & A'_{11} & A_{12}
\end{bmatrix}
\]

...(17)

Now, using algebra of logics, we obtain the following results:

\[
A'_1 = \begin{bmatrix}
x'_3 \\
x_3 \\
x'_9
\end{bmatrix}
\]
\[ A'_1 A'_2 = \begin{bmatrix} x'_3 \\ x'_4 \\ x'_9 \end{bmatrix} \begin{bmatrix} x_4 & x_9 \end{bmatrix} = \begin{bmatrix} x'_3 & x_4 & x_9 \end{bmatrix} \] ... (18)

Similarly,

\[ A'_1 A'_2 A'_3 = \begin{bmatrix} x'_3 & x'_4 & x_5 & x_9 \end{bmatrix} \] ... (19)

\[ A'_1 A'_2 A'_3 A'_4 = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x_6 & x_9 \end{bmatrix} \] ... (20)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x_7 & x_9 \end{bmatrix} \] ... (21)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x_8 & x_9 \end{bmatrix} \] ... (22)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x'_8 & x'_9 & x_{10} & x_{11} \end{bmatrix} \] ... (23)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 A'_8 = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x'_8 & x'_9 & x_{10} & x_{11} \end{bmatrix} \] ... (24)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 A'_8 A'_9 = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x'_8 & x'_9 & x_{10} & x_{11} \end{bmatrix} \] ... (25)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 A'_8 A'_9 A'_{10} = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x'_8 & x'_9 & x_{10} & x_{11} \end{bmatrix} \] ... (26)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 A'_8 A'_{10} A_{11} = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x'_8 & x'_9 & x_{10} & x_{11} \end{bmatrix} \] ... (27)

\[ A'_1 A'_2 A'_3 A'_4 A'_5 A'_6 A'_7 A'_8 A'_{10} A_{11} A_{12} = \begin{bmatrix} x'_3 & x'_4 & x'_5 & x'_6 & x'_7 & x'_8 & x'_9 & x_{10} & x_{11} \end{bmatrix} \] ... (28)

Using equations (19) through (28) in equation (17), we obtain:
Using (29), equation (2) gives the value of $F(x_1, x_2, \ldots, x_{11})$. This value is disjunction of pair-wise disjoint conjunctions, therefore the reliability of considered solar PV system is given by:

$$R_3 = \text{Pr}\{F(x_1, x_2, \ldots, x_{11}) = 1\}$$

$$= R_1 R_2 \{R_3 + S_3 R_4 + S_3 S_4 R_5 + S_3 S_4 S_5 R_6 + S_3 S_4 S_5 S_6 R_7 + S_3 S_4 S_5 S_6 S_7 R_8 \}$$

$$+ S_9 R_9 R_{11} \{R_3 S_4 S_5 S_6 S_7 S_8 + R_4 R_5 R_6 R_7 R_8 + R_4 S_3 S_5 S_6 S_7 S_8 \}$$

$$+ S_3 S_4 S_5 R_6 R_7 R_8 + S_3 S_4 S_5 S_6 S_7 S_8 + S_3 S_4 S_5 R_6 R_7 R_8 + S_3 S_4 S_5 S_6 S_7 S_8 \} \ldots(30)$$

where, $R_i$ is the reliability corresponding to system state $x_i$ and $S_i = 1 - R_i$, $\forall i = 1, 2, \ldots, 11$. 

$$f(x_3, x_4, \ldots, x_{11}) = \begin{bmatrix} x_3 & x_9 \\ x_3' & x_4 & x_9 \\ x_3' & x_4' & x_5 & x_9 \\ x_3' & x_4' & x_5' & x_6 & x_9 \\ x_3' & x_4' & x_5' & x_6' & x_7 & x_9 \\ x_3' & x_4' & x_5' & x_6' & x_7' & x_8 & x_9 \\ x_3' & x_4' & x_5' & x_6' & x_7' & x_8' & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3' & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{bmatrix} \ldots(29)$$
6. SOME PARTICULAR CASES

CASE I: When reliability of each component is $R$

In this case, $R_i (i = 1, 2, \ldots, 11) = R$ and by putting this in equation (30), we obtain

$$R_S = 6R^4 - 15R^3 + 26R^2 - 50R + 91R - 107R^0 + 56R^{10} - 11R^{11} + 5R^{12} \quad \ldots(31)$$

CASE II: When failures follow Weibull time distribution

Let $\lambda$ be the failure rate of each component of system and it follows weibull time distribution, then

$$R = \exp \left\{-\lambda t^\alpha \right\}$$

where $\alpha$ is a real positive parameter.

Putting this value in equation (31), we obtain

$$R_{SW}(t) = 6e^{-4\lambda t^\alpha} - 15e^{-5\lambda t^\alpha} + 26e^{-6\lambda t^\alpha} - 50e^{-7\lambda t^\alpha} + 91e^{-8\lambda t^\alpha} - 107e^{-9\lambda t^\alpha}$$

$$+ 56e^{-10\lambda t^\alpha} - 11e^{-11\lambda t^\alpha} + 5e^{-12\lambda t^\alpha} \quad \ldots(32)$$

CASE III: When failures follow exponential time distribution

Exponential time distribution is a particular case of Weibull time distribution for $\alpha = 1$ and is very useful in numerous practical problems. So, putting $\alpha = 1$ in equation (32), we get

$$R_{SE}(t) = 6e^{-4\lambda t} - 15e^{-5\lambda t} + 26e^{-6\lambda t} - 50e^{-7\lambda t} + 91e^{-8\lambda t} - 107e^{-9\lambda t}$$

$$+ 56e^{-10\lambda t} - 11e^{-11\lambda t} + 5e^{-12\lambda t} \quad \ldots(33)$$

Also, an important reliability parameter M.T.T.F., in this case, is given by

$$M.T.T.F. = \int_0^\infty R_{SE}(t)dt$$

$$= \frac{1}{\lambda} \left( \frac{3}{2} - 3 + \frac{13}{3} - \frac{50}{7} + \frac{91}{8} - \frac{107}{9} + \frac{56}{10} - 1 + \frac{5}{12} \right)$$

$$= \frac{0.19325396}{\lambda} \quad \ldots(34)$$

7. NUMERICAL COMPUTATION

For a numerical computation, consider the values:
(i) \( \lambda = 0.001, \alpha = 2 \) and \( t = 0,1,2,\ldots,10 \). Using these values in equation (32), we compute table-1.

(ii) \( \lambda = 0.001, \) and \( t = 0,1,2,\ldots,10 \). Using these values in equation (33), we compute table-1.

(iii) \( \lambda = 0,0.01,0.02,\ldots,0.10 \). Using these values in equation (34), we compute table-2.

8. RESULTS AND DISCUSSION

The graph of table-1 has shown in fig-2. Analysis of table-1 and fig-2 reveals that reliability function \( RSW(t) \) decreases catastrophically in the beginning but thereafter it decreases constantly. The value of \( RSE(t) \) remains better as compared of \( RSW(t) \).

The graph of table-2 has shown in fig-3. A critical examination of table-2 and fig-3 yields that the value of MTTF decreases rapidly as we make increase in the values of failure rate \( \lambda \) but thereafter it decreases in a constant manner.

9. FIGURES AND TABLES

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Table-1
Fig-1

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Table-2
10. REFERENCES