ELECTRIC FIELD EFFECT ON THE ACOUSTICALLY DRIVEN SEMICONDUCTOR SUPERLATTICES

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ABSTRACT
We study the effect of applying a static electric field to the acoustically driven semiconductor superlattices. Semiconductor superlattices have a large effective lattice period of ~ 1-10 nanometres. When an electric field is applied along its axis the electron starts to oscillate. The frequency range of this oscillation can be as large as terahertz. The oscillation causes localisation of electron trajectories as the field increases. It had also been shown that electron oscillations are induced in superlattices when acoustic wave is applied. The electron oscillations depend critically on the acoustic wave amplitude, whether it is less or greater than the critical value.

In this work, when both the acoustic wave and electric field were applied together, we obtained the peak drift velocity significantly higher compared to when the acoustic wave amplitude or electric field was applied alone. We analysed numerically computed electron trajectories for different values of the acoustic wave amplitude and the electric field applied imparting the high drift velocity.

1. Introduction
Semiconductor superlattices were proposed in the 1970’s by Esaki and Tsu. It is a periodic crystal lattice of two or more semiconductors. The semiconductors will have similar lattice constants but different band gaps deposited alternately on each other to form the superlattices [1]. The different band structure thus creates a periodic potential along the growth axis to form this artificial crystal. During the growing process the superlattices period can be controlled to have larger period than the constituent materials [2]. An example of superlattices of two semiconductors layers deposited alternately to form a nanostructure is GaAs/AlAs. The interest in the superlattices is growing rapidly due to nonlinear phenomena generated in a semiconductor superlattices miniband. The nonlinear phenomena are very useful application in the ultrafast electronics industry.

It had been observed that when static electric field is applied along the axis of the superlattices, Bloch oscillation occurs as the electric field is increasing [3] and negative differential velocity occurred as the field increases further. When acoustic wave amplitude is applied along the superlattices axis, high frequency oscillations occurred [4]. Two distinct regimes are observed depending on the wave amplitude of the acoustic wave. The electron is dragged in the superlattices when the wave amplitude is less than the critical value $U_c$. The critical value depends on the parameter of the semiconductor superlattices. As soon as the wave amplitude increases above this critical value, we have high frequency current oscillation and then Bloch oscillation [5, 6].

In this work, static electric field was applied along the axis of acoustically driven superlattices, Bloch oscillation occurs which causes the localisation of electron trajectories and form region of negative differential velocity as we increased the external fields further. The large effective lattice period of the semiconductor superlattices increases the frequency of the Bloch oscillation thereby allowing the electron to Bloch oscillates before scattering. For Bloch oscillation to occur the field must be strong and the crystal should have a large period, which is found in the superlattices. In ordinary crystals, electron cannot get to the zone boundary because of the limited time of electron collisions that is about $10^{-14}$ sec.

2. Model of electron dynamics
In the tight-binding approximation, the kinetic energy versus crystal momentum dispersion relation for the miniband propagating along $x$-axis is given by,

$$E (P_x) = \frac{\Delta}{2} \left[ 1 - \cos \frac{P_x d}{\hbar} \right]$$

the kinetic energy and the Potential energy of a longitudinal acoustic wave propagating along $x$-axis of the superlattices is described as
\[ V(x, t) = -U \sin[k_s(x) - w_s t] \]  

(2)

This was derived by taking the potential energy due to the strain of the lattice, \( S \) which is \(-S_0 \sin(k_s(x) - w_s t)\) and the wave amplitude, \( U = DS_0 \) where \( D \) is the electron-phonon coupling constant \([7]\). Experimentally, it is found to be \(~10 \text{ eV}\). For this system, the Hamiltonian when acoustic wave and electric field are applied along the axis of superlattices with potential energy of \( eF x \) for the static electric field is therefore

\[ H(x, p_x) = \frac{\hbar}{2} \left[ 1 - \cos\frac{p_x d}{\hbar} \right] - U \sin[k_s(x) - w_s t] + eF x \]  

(3)

The semi classical equations of electron motion for this system will then be Hamilton’s equation described as

\[
\begin{align*}
v_x &= \frac{dx}{dt} = \frac{\partial H}{\partial p_x} = \frac{\Delta d}{2\hbar} \sin\frac{p_x d}{\hbar} \\
\frac{dp_x}{dx} &= \frac{\partial H}{\partial x} = k_s U \cos[k_s(x) - w_s t] + eF
\end{align*}
\]  

(4)

(5)

The electron trajectories are therefore be obtained by solving the above equations numerically using Runge Kutta algorithms with initial conditions that \( x = 0 \) at \( t = 0 \) and assuming there is no scattering. The lattice period, which is larger in the superlattices, will allow electron to traverse a whole minizone even if scattering will occur \([8]\).

3. Results and Discussion

Minibands in the superlattices allow electrons to perform terahertz frequency Bloch oscillations when high external field is applied along the axis of superlattice. The figures below showed the electron dynamics for acoustically driven superlattice in the electric field. We calculate the electron trajectories by solving equation (4) and (5) numerically for different wave amplitude and electric field. It was observed that at low values of wave amplitude and electric field the usual dragging regime that was observed when acoustic wave alone was applied along the superlattices axis was also observed. When static electric field was applied to the acoustically driven superlattices it induced charge domains, which interact with the dragging charge in the lattice and produce frequency current oscillation. As the external fields are increased further we observed high frequency current oscillation as shown in Figures.
Figure 1: Electron trajectory $x(t)$ for $U = 1\text{meV}$ and $F = 1 \times 10^2 \text{V/m}^{-1}$

Figure 1 showed the electron trajectory in real space, where electron is confined within a single potential well in the acoustic wave and it oscillate back and forth across the well. As we increased both the acoustic wave and electric field, we observed high frequency as shown in Figure 2 and the electrons begin to Bloch oscillate.

Figure 2: Electron trajectory $x(t)$, when static electric field and energy amplitude of acoustic wave were applied together along the axis of semiconductor superlattices. $F = 5 \times 10^5 \text{V/m}^{-1}$ and $U = 4\text{meV}$. 
In Figure 3 the Bloch-like oscillations caused by the acoustic wave was interrupted by jumps in the negative direction. While for a very high acoustic wave and electric field the jump was increased and in negative direction as shown in Figure 4.

To determine the measured transport characteristics of electron in superlattices we calculate drift velocity of electron, using the Esaki-Tsu model. The model described electrons behaviour in the superlattices [1], electron moving under the influence of acoustic wave and static electric field in the $x$-direction with a cosine dispersion relation, will have drift velocity of the form
\[ v_d = \frac{1}{\tau} \int_0^\infty \dot{x}(t)e^{-t/\tau} \]  

(6)

\( \tau \) is the scattering time.

Figure 5: The graph of \( v_d(U) \) (dash curve) when acoustic wave alone was applied to the superlattices and (solid curve) compare when the miniband electron was driven by both electric field and acoustic wave.

Figure 5 shows an overshoot of drift velocity and a large value of negative differential velocity, the resulting curve \( v_d(U, F) \) (Fig. 5) showed for different strengths of the energy amplitude and the static electric field. The combination of these external fields gives peak value of drift velocity even when \( U \leq U_c \) before the electron entered a region where the velocity starts to drop. The \( v_d(U, F) \) curve passes a maximum and decreases with increasing external fields as more and more electrons complete Bloch oscillations before scattering. The colour map of \( v_d(U, F) \) (Fig. 6) explain this further by showing the point of highest drift velocity when \( U \leq U_c \) and \( F \leq F_c \), where \( U_c \) and \( F_c \) are the critical values whose values depend on the parameter of the semiconductor superlattices. The current density,

\[ j = env_d \]

So that the conductivity,

\[ \sigma = \frac{\partial j}{\partial F} \]

\[ = en \frac{\partial v_d}{\partial F} \]
Figure 6: Colour map of $v_d(U, F)$. The drift velocity versus $U$ and $F$. 

Figure 6 showed the dependence of drift velocity on the combination of the electric field and wave amplitude and this gives a deeper insight into joint effect of both on the dynamics of electron. We observed a maximum drift velocity at the critical value of both electric field and acoustic wave. A critical value is the value of the field when dynamical change occurred.

4. CONCLUSION

The static electric field applied to the acoustically driven electron in semiconductor superlattices produce an interesting result. The charge domains induced by the static electric field interact with the charges in the lattice to produced high frequency current oscillation. A high drift velocity obtained implies that we have higher fundamental frequency of the current self-oscillations at small values of static electric field and acoustic wave combined. When applied field both electric field and acoustic wave is small, conductivity is high that is $\sigma > 0$ and as we increased the applied field we have negative differential conductivity which makes the conductivity to decrease, $\sigma < 0$.

5. REFERENCES