AN ORTHOTROPIC ADAPTIVE SHALLOW CYLINDRICAL SHELL ON ELASTIC FOUNDATION

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ABSTRACT

With growth and emergence in the field of adaptive materials, the need arises to study their applications in the field of structural, aerodynamic, aerospace and other fields. These materials can be used as sensors, transducers, and actuators. Although their basic constitutive relations are already developed, but there is still a great deal of scope left in the field of applications. With this aim, a nonlinear static analysis of orthotropic piezoelectric shallow cylindrical shell on Pasternak foundation is investigated in the present work. Basic formulation of the problem is based on strain energy concept, and the governing differential equations are obtained by using Euler’s variational principle. Galerkin error minimization technique has been used to solve the governing differential equations. The results are presented for simply supported immovable edge boundary condition. Influences of shell geometry, foundation parameter, and piezoelectric properties on load–deflection characteristics for different radius-to-thickness ratios are studied. Numerical results have been obtained for different values of geometrical parameters in terms of load, displacement, and electric potential. Geometrical parameters are represented through non-dimensional entities $\eta = a^2/Rh$, $\lambda = K_a^2/D_{11}$, and $\mu = G_a^2/D_{11}$. The results are compared with nonlinear static analysis of an orthotropic shallow cylindrical shell without piezoelectric layer on Pasternak foundation. It is observed that an increase in the value of piezoelectric constant decreases the deflection of the shallow cylindrical shell under the identical values.

Keywords: Orthotropic; piezoelectric; shallow cylindrical shell; elastic foundation; nonlinear static analysis.

1. INTRODUCTION

Structural engineering and material science have entered into a new age bought about by the development of adaptive materials and their applications in the intelligent structures. Piezoelectric materials provide a transformation between electric and mechanical energy, and can thus be used for mechanical and acoustic input and output devices. Owing to this ability to provide sensor (input) and actuator (output) functions at the same time, piezoelectric belong to the materials family that is often referred to as 'intelligent materials'. Present study is concentrated on studies of shallow shells .A shell is usually regarded as being shallow if $\frac{l_{\text{min}}}{f} \geq 5$ and $\frac{R_{\text{min}}}{h} \geq 5$, where $l_{\text{min}}$ is minimum dimension of the shell in plan and ‘f’ is sag, the highest elevation above the base.

Foundation is a load-carrying member. Successful design of structures involving foundation- structure interaction depends to a large extent upon the use of appropriate foundation model. There are various elastic foundation models (Fig.1a-b-c), out of which the present study is concentrated on Pasternak foundation. Pasternak model is a biparametric elastic foundation model characterized by two independent elastic constants. It is suitable for homogeneous soil deposits and rocky soils.

The strain energy equation of the Pasternak foundation can be expressed in the terms foundation modulii as.

$$ U = \frac{1}{2} \int \left\{ k_1 w^2 + G \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \right\} da $$
where $k_1$ = modulus of sub grade reaction for the foundation and $G$ = shear modulus of the sub grade. If it is viewed as a constant tension, it becomes the Pasternak modulus, and if $G = 0$, the modulus reduces to the Winker Pasternak foundation.

2. LITERATURE REVIEW AND OBJECTIVE OF PRESENT WORK

Investigations carried out in the past [1, 2] were limited to the cases of non-piezoelectric shallow cylindrical shells. The works presented in [3-5] were restricted to cylindrical shells only and no consideration was made for shallow cylindrical shells. Based on the literature survey, it can be inferred that a lot of work has been done in the field of shells on elastic foundation as well as on piezoelectric shells, but not on shallow cylindrical shell on elastic foundation. Piezoelectric shell in cylindrical bending has also been extensively investigated, but only for the case of axi-symmetric loading.

In case of shells on elastic foundation, the loading and deformation are not symmetrical along the circumferential direction. The objective of this study is to investigate the nonlinear static behavior of orthotropic shallow cylindrical shells on a Pasternak type elastic foundation and study the influence of various shell and foundation parameters on load-deflection as well as load-electric potential characteristics.

3. FORMULATION OF THE PROBLEM

We consider the large symmetric deflection of a shallow cylindrical shell subjected to uniform normal pressure $q$ on its concave surface and resting on elastic foundation. Coordinate system employed is shown in Fig. 1. For circular cylindrical shell the distance of the middle surface of the shell is given by

$$z = \frac{a^2}{2R} \left(1 - \frac{x^2}{a^2}\right)$$

The assumptions made in this analysis are (1) The shell is orthotropic shallow cylindrical without initial imperfections (2) It is assumed that the piezoelectric material is incorporated into the matrix. (3) Thickness of
the shell is much less than other dimensions of the shell. (4) Material of the shell is assumed to be homogeneous and orthotropic. (5) The shell is initially subjected to axisymmetric external pressure. (6) Electric field is considered varying linearly in radial direction. (7) Body forces are assumed to be zero.

Constitutive equations for the piezoelectric material are given by

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix}
= 
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
 c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
 c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\
 c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\
 c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\
 c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{12} \\
e_{13} \\
e_{22} \\
e_{23} \\
e_{31}
\end{bmatrix}
+ 
\begin{bmatrix}
e_{11} \\
e_{12} \\
e_{13} \\
e_{22} \\
e_{23} \\
e_{31}
\end{bmatrix}
= 
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\tag{2}
\]

where \(\sigma\) are the stresses, \(c\) are elastic coefficients, \(\varepsilon\) are strains, \(e\) are piezoelectric stress coefficients and \(E\) are electric field vector.

The relations between electric displacement and electric field are given by

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
= 
\begin{bmatrix}
e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\
e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\
e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{23} \\
\varepsilon_{31}
\end{bmatrix}
+ 
\begin{bmatrix}
K_{11} & K_{12} & K_{13} \\
K_{21} & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\tag{3}
\]

where \(D\) are electric displacement vector and \(k\) are dielectric coefficients.

### 3.1 SIMPLIFICATION TO 2-DIMENSIONAL PLANE STRESS CONDITION

As per the conditions of the problem, the formulation is reduced to a 2-dimensional plane stress condition. Hence

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix}
= 
\begin{bmatrix}
c_{11} & c_{12} & 0 \\
 c_{21} & c_{22} & 0 \\
 0 & 0 & c_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{12} \\
e_{12}
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & e_{31} \\
0 & 0 & e_{32} \\
0 & 0 & e_{36}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\tag{4}
\]
The normal forces \( N \) and moment \( M \) per unit length are obtained by
\[
\begin{align*}
\int \sigma \, dz & = N \\
\int z \sigma \, dz & = M
\end{align*}
\]  
(6)

If the directions ‘1’ and ‘2’ are represented by coordinate axes ‘x’ and ‘y’ respectively, then \([N]\) and \([M]\) are given by
\[
\begin{align*}
N_{x} &= \begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \end{bmatrix} + \begin{bmatrix} k_{11} \end{bmatrix} + h \begin{bmatrix} E_{x} \end{bmatrix} \\
N_{y} &= \begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \end{bmatrix} + \begin{bmatrix} k_{22} \end{bmatrix} + h \begin{bmatrix} E_{y} \end{bmatrix} \\
N_{xy} &= \begin{bmatrix} 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \end{bmatrix} + \begin{bmatrix} 0 & 0 & k_{33} \end{bmatrix} + \begin{bmatrix} E_{z} \end{bmatrix}
\end{align*}
\]  
\[\begin{bmatrix} M_{x} \\ M_{y} \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 0 & 0 & D_{11} & D_{12} & 0 & k_{xx} \\ 0 & 0 & D_{12} & D_{22} & 0 & k_{yy} \\ 0 & 0 & 0 & 0 & D_{66} & k_{xy} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \\ \varepsilon_{yy} \end{bmatrix} + h \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} E_{z} \end{bmatrix}
\]  
(7)

The relation between strain and mechanical displacements are written as
\[
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial z}{\partial x} \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial z}{\partial y} \\
\varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}
\end{align*}
\]  
(9a-c)

The curvature displacement relation are expressed as
\[
\begin{align*}
k_{xx} &= \frac{\partial^2 w}{\partial x^2}, \quad k_{yy} = \frac{\partial^2 w}{\partial y^2}, \quad k_{xy} = \frac{\partial^2 w}{\partial x \partial y}
\end{align*}
\]  
(10a-c)

As the total energy of the system is equal to the sum of strain energies due to extension, bending, piezoelectric effect and deformation of the foundation; hence it is expressed as
\[
U = \frac{1}{2} \left( N_{x} \varepsilon_{xx} + N_{y} \varepsilon_{yy} + N_{xy} \varepsilon_{xy} + M_{x} k_{xx} + M_{y} k_{yy} + M_{xy} k_{xy} + D_{xx} E_{x} + D_{yy} E_{y} + D_{xy} E_{z} +
\right)
\]  
\[\int k \mu \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) \, dx \, dy
\]  
(11)
\[ U = \frac{1}{2} \left[ (A_1 \varepsilon_{xx} + A_2 \varepsilon_{yy} + h \varepsilon_{31} E_3) \varepsilon_{xx} + (A_2 \varepsilon_{xx} + A_2 \varepsilon_{yy} + h \varepsilon_{32} E_2) \varepsilon_{yy} + (A_6 \varepsilon_{xy} + h \varepsilon_{36} E_3) \varepsilon_{xy} \right. \\
+ \left. (D_{11} k_{xx} + D_{12} k_{yy}) k_{xx} + (D_{12} k_{xx} + D_{22} k_{yy}) k_{yy} + (D_{66} k_{xy}) k_{xy} + h k_{11} E_x^2 + h k_{22} E_y^2 + h(e_{31} \varepsilon_{xx} + e_{32} \varepsilon_{yy} + e_{36} \varepsilon_{xy}) E_z k w^2 - 2qw + G \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] dx dy \]

With the help of Eq. (7), the Eq. (11) becomes

\[ U = \frac{1}{2} \left[ (A_1 \varepsilon_{xx} + A_2 \varepsilon_{yy} + h \varepsilon_{31} E_3) \varepsilon_{xx} + (A_2 \varepsilon_{xx} + A_2 \varepsilon_{yy} + h \varepsilon_{32} E_2) \varepsilon_{yy} + (D_{11} k_{xx} + D_{12} k_{yy}) k_{xx} + \right. \\
+ \left. (D_{12} k_{xx} + D_{22} k_{yy}) k_{yy} + (D_{66} k_{xy}) k_{xy} + (h \varepsilon_{31} \varepsilon_{xx} + h \varepsilon_{32} \varepsilon_{yy}) E_z + h \varepsilon_{36} \varepsilon_{xy} E_z + \right. \\
+ \left. \frac{1}{2} \left[ h k_{11} E_x^2 + h k_{22} E_y^2 + h k_{33} E_z^2 \right] \right] + \frac{1}{2} \left[ k w^2 - 2qw + G \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \right] dx dy \]

(12)

where

\[ E_x = \frac{\partial \phi}{\partial x}, \quad E_y = \frac{\partial \phi}{\partial y}, \quad \text{and} \quad E_z = \frac{\partial \phi}{\partial z} = \frac{\phi}{z} \]

Since we know that for a shallow circular shell, the geometry is

\[ z = \frac{a^2}{2R} \left( 1 - \frac{x^2}{a^2} \right), \quad \text{therefore} \quad \frac{\partial z}{\partial x} = \frac{a}{2R} \left( \frac{2x}{a} \right) = \frac{x}{R} \quad \text{and} \quad \frac{\partial z}{\partial y} = 0 \]

Using Euler’s variation principle:

\[ \frac{\partial F}{\partial u} + \frac{\partial F}{\partial E_x} \frac{\partial E_x}{\partial x} + \frac{\partial F}{\partial E_y} \frac{\partial E_y}{\partial y} = 0 \]  

(13)

\[ \frac{\partial F}{\partial v} + \frac{\partial F}{\partial E_x} \frac{\partial E_x}{\partial x} + \frac{\partial F}{\partial E_y} \frac{\partial E_y}{\partial y} = 0 \]  

(14)

\[ \frac{\partial F}{\partial w} + \frac{\partial F}{\partial E_x} \frac{\partial E_x}{\partial x} + \frac{\partial F}{\partial E_y} \frac{\partial E_y}{\partial y} = 0 \]  

(15)

and applying Eq. (13) to integrand in Eq. (12), we get

\[ 0 = \frac{\partial}{\partial x} \left[ \frac{12}{h^2} (2D_{11} \varepsilon_{xx} + 2D_{12} \varepsilon_{yy}) \right] - \frac{\partial}{\partial y} (8D_{66} \varepsilon_{xy}) - \frac{\partial}{\partial x} (e_{31} \phi) - \frac{\partial}{\partial y} (e_{36} \phi) = 0 \]

On substituting the strain terms given by Eqs. (9a-b-c) in the Eq. (17), we get
Similarly Eqs (14, 15, 16) are applied to integrand in Eq (12), which yield Eqs. (19a), (20a) and (21a). Further on substituting the strain terms given by Eqns. (9a-b-c) in Eqs. (19a), (20a), (21a); we get Eqs. (19b), (20b), and (21b). These are given below.

\[
0 - \frac{\partial}{\partial y} \left[ \frac{12}{h^2} \left( 2D_{12} \varepsilon_{xx} + 2D_{22} \varepsilon_{yy} \right) \right] - \frac{\partial}{\partial y} \left[ \frac{12}{h^2} \left( 2D_{12} \varepsilon_{x\alpha} + 2D_{22} \varepsilon_{y\alpha} \right) \right] - \frac{\partial}{\partial y} \left( e_{31} \phi \right) - \frac{\partial}{\partial y} \left( e_{36} \phi \right) = 0
\]  

\[
2k_w - 2q \left[ \frac{12}{h^2} \left( D_{12} \varepsilon_{x\alpha} \frac{\partial w}{\partial \alpha} + 2D_{12} \varepsilon_{y\alpha} \frac{\partial \varepsilon_{y\alpha}}{\partial \alpha} + 8D_{12} \varepsilon_{y\alpha} \frac{\partial \varepsilon_{y\alpha}}{\partial \alpha} \right) \right] + 8G \frac{\partial w}{\partial y} + 0
\]

\[
2k_w - 2q \left[ \frac{12}{h^2} \left( D_{12} \varepsilon_{x\alpha} \frac{\partial w}{\partial \alpha} + 2D_{12} \varepsilon_{y\alpha} \frac{\partial \varepsilon_{y\alpha}}{\partial \alpha} + 8D_{12} \varepsilon_{y\alpha} \frac{\partial \varepsilon_{y\alpha}}{\partial \alpha} \right) \right] + 8G \frac{\partial w}{\partial y}
\]

\[
\varepsilon_{31} \varepsilon_{xx} + e_{32} \varepsilon_{yy} + e_{36} \varepsilon_{xy} + \frac{K_{13}}{h} \phi \cdot \frac{\partial}{\partial x} \left( h \kappa_{11} \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial y} \left( h \kappa_{22} \frac{\partial \phi}{\partial y} \right) \frac{e_{31}}{R} = 0
\]
4. SOLUTION OF GOVERNING EQUATIONS

Thus, on applying the conditions of extremum, four equations in four variables are obtained. In order to solve these equations, Galerkin-error minimizing technique is used. The approximate solutions are assumed according to the boundary conditions present at the ends. For the simply supported immovable end conditions the \( u, v, w \) and \( \phi \) are assumed to be the functions of \( x \) and \( y \) as follows.

\[
\begin{align*}
\phi &= \phi_0 \sin \left( \frac{\pi x}{2a} \right) \sin \left( \frac{\pi y}{2b} \right) \\
0.22 && 0.22
\end{align*}
\]

On recalling Eq. (18) and expanding the derivation terms, we get coupled equations. Similar equations are obtained from Eqs. (19b), (20b) and (21b). These are given as follows.

APPENDIX I

Now recalling Eqn. (18), we write

\[
- \frac{e_{11}}{R} \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x \partial y} \right) + e_{31} \left( \frac{\partial^2 v}{\partial y^2} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y \partial x} \right) + e_{31} \left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial x} \right) + e_{33} \left( \frac{\partial^2 w}{\partial x \partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y \partial x} \right) + e_{31} \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x \partial y} \right) + e_{31} \left( \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y \partial x} \right) + e_{33} \left( \frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y \partial x} \right) + e_{31} \left( \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} \right) + e_{33} \left( \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial y} \right) = 0
\]

Further on expanding the derivation terms, we get

\[
- \frac{1}{h^2} \left( 2. D_{11} \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x \partial y} \right) + 2. D_{12} \left( \frac{\partial^2 v}{\partial y^2} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y \partial x} \right) + 2. D_{13} \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x \partial y} \right) + 2. D_{21} \left( \frac{\partial^2 v}{\partial y^2} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y \partial x} \right) + 2. D_{22} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y \partial x} \right) + 2. D_{23} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y \partial x} \right) + 2. D_{31} \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x \partial y} \right) + 2. D_{32} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y \partial x} \right) + 2. D_{33} \left( \frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x \partial y} \right) + 2. D_{41} \left( \frac{\partial^2 v}{\partial y^2} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y \partial x} \right) + 2. D_{42} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial y \partial x} \right) + 2. D_{43} \left( \frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x \partial y} \right) \right) - \frac{\partial}{\partial x} (\epsilon_{1x} \phi) = 0
\]
On putting the values of derivative terms in the above expression, we obtain

\[
\begin{align*}
-\frac{12}{h^2} & \left[ 2D_{11} \left\{ \frac{\pi^2 u_x \sin \frac{\pi x}{2a}}{4a^2} \sin \frac{\pi y}{2b} + \left( \frac{\pi w_y \cos \frac{\pi x}{2a}}{2a} \sin \frac{\pi y}{2b} \right) \right\} \\
& - \left( \frac{\pi^2 w_y \sin \frac{\pi x}{2a}}{4a^2} \sin \frac{\pi y}{2b} \right) \right] + \left( \frac{\pi^2 w_y \sin \frac{\pi x}{2a}}{4a^2} \sin \frac{\pi y}{2b} \right) + \\
& \left( - \frac{1}{R} \left( \frac{\pi w_x \cos \frac{\pi x}{2a}}{2a} \sin \frac{\pi y}{2b} \right) \right) + 2D_{12} \left\{ \frac{\pi w_x \cos \frac{\pi x}{2a}}{4ab} \cos \frac{\pi y}{2b} \right\} \right] - 8D_{10} \cdot \\
& \left( \frac{\sin \frac{\pi y}{2b}}{2b} \right) + \left( \frac{\pi w_x \sin \frac{\pi x}{2a}}{4a^2} \sin \frac{\pi y}{2b} \right) + \left( \frac{\pi w_x \sin \frac{\pi x}{2a}}{4a^2} \cos \frac{\pi y}{2b} \right) \\
& \left( \frac{\pi w_y \cos \frac{\pi x}{2a}}{4ab} \cos \frac{\pi y}{2b} \right) \right) \right] - 8D_{10} \cdot \\
& \left( \frac{\cos \frac{\pi y}{2b}}{2b} \right) = 0
\end{align*}
\]

Similarly, from Eqn. (19b) we get,

\[
\begin{align*}
-\frac{12}{h^2} & \left[ 2D_{11} \left\{ \frac{\pi^2 u_x \cos \frac{\pi x}{2a}}{4ab} \cos \frac{\pi y}{2b} + \left( \frac{\pi w_y \cos \frac{\pi x}{2a}}{2a} \sin \frac{\pi y}{2b} \right) \right\} \\
& - \left( \frac{\pi^2 w_y \cos \frac{\pi x}{2a}}{4ab} \cos \frac{\pi y}{2b} \right) \right] + \left( \frac{\pi^2 w_y \cos \frac{\pi x}{2a}}{4ab} \cos \frac{\pi y}{2b} \right) + \\
& \left( - \frac{1}{R} \left( \frac{\pi w_x \cos \frac{\pi x}{2a}}{2ab} \cos \frac{\pi y}{2b} \right) \right) + 2D_{12} \left\{ \frac{\pi w_x \cos \frac{\pi x}{2a}}{4ab} \cos \frac{\pi y}{2b} \right\} \right] + 0
\end{align*}
\]
Using Galerkin error minimizing technique the value of $\phi_0$ for square panel (i.e. $a = b$), is found in terms of $w_0$ as

$$\phi_0 = \frac{2 \left( 9a^2 \cdot h \cdot e_{31} \cdot w_0 - 8h \cdot R \cdot e_{31} w_0^2 - 16h \cdot R \cdot e_{36} w_0^2 \right)}{9R \left( h^2 \pi^2 \kappa_{11} + h^2 \pi^2 \kappa_{22} + 4a^2 \kappa_{33} \right)}$$

and as $e_{31} = e_{32}$, therefore

$$\phi_0 = \frac{2 \left( 9a^2 \cdot h \cdot e_{31} \cdot w_0 - 16h \cdot R \cdot e_{31} w_0^2 \right)}{9R \left( h^2 \pi^2 \kappa_{11} + h^2 \pi^2 \kappa_{22} + 4a^2 \kappa_{33} \right)}$$

It can be seen that Eq (23) gives a direct relation between $\phi_0$ and $w_0$. Thus for a deformed shape/profile the potential can be directly obtained.

5. RESULTS AND DISCUSSION

Numerical results have been obtained for different values of geometrical parameter in terms of load, displacement and electric potential. Geometrical parameters are represented through non-dimensional entities $\eta = a^2/Rh$, $\lambda = ka^4/D_{11}$ and $\mu = Ga^2/D_{11}$. Values of material properties are as follows.

- $D_{11} = 1$ Nm,
- $D_{22} = 0.785$ Nm,
- $D_{33} = 0.463$ Nm,
- $\nu_1 = 0.089$, $\nu_2 = 0.07$, $\alpha = \sqrt{a}$ m,
- $h = 0.1$m,
- $\kappa_{11} = 1.531 \times 10^8$ F/m,
- $\kappa_{22} = 1.531 \times 10^8$ F/m,
- $\kappa_{33} = 1.5045 \times 10^8$ F/m

Following values of non-geometrical parameters are considered:

- $\eta = 0.25, 1.25, 2.5$, $\lambda = 20, 50, 100, 150$,
5.1 EFFECT OF GEOMETRICAL PARAMETER $\eta$

The effects of variation in $\eta$ on load-deflection response are depicted in Table 1 and Fig. 3.

Table 1. Load-deflection variations for simply supported piezoelectric shell for different values of $\eta$

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_0/h$ for $\eta = 0.25$</th>
<th>$w_0/h$ for $\eta = 1.25$</th>
<th>$w_0/h$ for $\eta = 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.831</td>
<td>0.705</td>
<td>0.530</td>
</tr>
<tr>
<td>200</td>
<td>1.260</td>
<td>1.103</td>
<td>0.880</td>
</tr>
<tr>
<td>300</td>
<td>1.552</td>
<td>1.383</td>
<td>1.143</td>
</tr>
<tr>
<td>400</td>
<td>1.778</td>
<td>1.604</td>
<td>1.355</td>
</tr>
<tr>
<td>500</td>
<td>1.966</td>
<td>1.789</td>
<td>1.535</td>
</tr>
</tbody>
</table>

The variation in the values of $w_0/h$ is shown as a function of non-dimensional load $Q$ for different values of $\eta$. It shows a decreasing trend in non-dimensional displacement $w_0/h$ with increase in the value of $\eta$. At $Q=500$, there is 21.943% decrease in $w_0/h$ when the value of $\eta$ is increased from 0.25 to 2.5.

5.2 EFFECT OF PIEZOELECTRIC PROPERTIES.

Different values of displacement are plotted against $Q$ for piezoelectric shell as well as for the shell without piezoelectric properties for $\eta=0.25$. As is evident from Table 2 and Fig. 4, there is a decrease in the value of $w_0/h$ with the application of piezoelectric properties to the shell. The decrease is, however, very small. It is about 1.14% only at $Q=500$. This can be attributed to very small values of piezoelectric constants.

Table 2. Effect on deflection for shell with piezoelectric properties.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_0/h$ for simple orthotropic shell</th>
<th>$w_0/h$ for piezoelectric shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.831</td>
<td>0.826</td>
</tr>
<tr>
<td>200</td>
<td>1.260</td>
<td>1.248</td>
</tr>
<tr>
<td>300</td>
<td>1.552</td>
<td>1.536</td>
</tr>
<tr>
<td>400</td>
<td>1.778</td>
<td>1.759</td>
</tr>
<tr>
<td>500</td>
<td>1.966</td>
<td>1.944</td>
</tr>
</tbody>
</table>
5.3 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS.

Table 3. Load-deflection variations for simply supported piezoelectric shell for different values of $e_{31}$.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-12}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-8}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.831</td>
<td>0.831</td>
<td>0.826</td>
</tr>
<tr>
<td>200</td>
<td>1.260</td>
<td>1.260</td>
<td>1.248</td>
</tr>
<tr>
<td>300</td>
<td>1.552</td>
<td>1.552</td>
<td>1.536</td>
</tr>
<tr>
<td>400</td>
<td>1.778</td>
<td>1.778</td>
<td>1.759</td>
</tr>
<tr>
<td>500</td>
<td>1.966</td>
<td>1.966</td>
<td>1.944</td>
</tr>
</tbody>
</table>
Variation in the piezoelectric constants $e_{31}$ and $e_{36}$ shows same behaviour as in case of $\lambda$ as shown in fig. 4.3.3. By the application of piezoelectric constant $e_{31} = -171 \times 10^{-12}$ there is a negligible decrease in the values of $w_0/h$ corresponding to $Q$, about $10^{-7}$% at $Q = 500$. When the constant is increased to $e_{31} = -171 \times 10^{-8}$, the change is still negligible. The increase to the value of $e_{31} = -171 \times 10^{-5}$ shows the decrease of about $1.0919\%$. The small decrease in the values of $w_0/h$ may be attributed to the very small values of piezoelectric constants that are taken as compared to the values of PZT-5A.

5.4 EFFECT OF NON-DIMENSIONAL FOUNDATION PARAMETER $\lambda$

Effects of variation in $\lambda$ on load-deflection response of the shallow cylindrical shell are presented in Table 4 and Fig. 6.

Table 4. Load-deflection variation for simply supported piezoelectric shell as a function of different values of $\lambda$.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_0/h$ for $\lambda = 20$</th>
<th>$w_0/h$ for $\lambda = 5$</th>
<th>$w_0/h$ for $\lambda = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.831</td>
<td>0.750</td>
<td>0.546</td>
</tr>
<tr>
<td>200</td>
<td>1.260</td>
<td>1.182</td>
<td>0.952</td>
</tr>
<tr>
<td>300</td>
<td>1.552</td>
<td>1.480</td>
<td>1.257</td>
</tr>
<tr>
<td>400</td>
<td>1.778</td>
<td>1.711</td>
<td>1.499</td>
</tr>
<tr>
<td>500</td>
<td>1.966</td>
<td>1.903</td>
<td>1.701</td>
</tr>
</tbody>
</table>
Fig 6. Load- deflection curves for simply supported piezoelectric shell for different values of $\lambda$.

The variation in the values of $w_0/h$ is shown as a function of non-dimensional load $Q$ for different values of $\lambda$. It is shown that with an increase in the value of $\lambda$ from 20 to 50, the value of $w_0/h$ decreases by 3.19 % at $Q=500$ and 13.48 % at $Q=1000$.

5.5 EFFECT OF PIEZOELECTRIC PROPERTIES.

Similarly the effects of piezoelectric properties are shown in Table 5 and Fig 7. Different values of displacement are plotted against $Q$ for piezoelectric shell as well as shell without piezoelectric properties, for constant $e_0=-17\times10^{-12}$. It is evident from the Fig. 1 and Table 1 that there is a decrease in the value of $w_0/h$ with the application of piezoelectric properties to the shell. The decrease is about 36.97 % at $Q=500$.

Table 5. Load-deflection variations for simply supported orthotropic shell and piezoelectric shell.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_0/h$ for non-piezoelectric shell</th>
<th>$w_0/h$ for piezoelectric shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.750</td>
<td>0.571</td>
</tr>
<tr>
<td>200</td>
<td>1.182</td>
<td>0.811</td>
</tr>
<tr>
<td>300</td>
<td>1.480</td>
<td>0.971</td>
</tr>
<tr>
<td>400</td>
<td>1.711</td>
<td>1.096</td>
</tr>
<tr>
<td>500</td>
<td>1.903</td>
<td>1.200</td>
</tr>
</tbody>
</table>
6 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS.

Variation in piezoelectric constant $e_{31}$ presents very interesting results as shown in Fig. 8 and Table 6. When the piezoelectric constant $e_{31} = -171 \times 10^{-12}$, there is a negligible decrease in the value of $\frac{w_0}{h}$. When this constant is increased to $e_{31} = -171 \times 10^{-8}$, the change is still negligible. But, when the value of $e_{31} = -171 \times 10^{-4}$, there is an unexpected decrease in the values of $\frac{w_0}{h}$. It is about 36.97% at $Q=500$.

Table 6. Load- deflection variations for different values of $e_{31}$

<table>
<thead>
<tr>
<th>Q</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-12}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-8}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-3}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.750</td>
<td>0.751</td>
<td>0.746</td>
<td>0.572</td>
</tr>
<tr>
<td>200</td>
<td>1.182</td>
<td>1.183</td>
<td>1.173</td>
<td>0.811</td>
</tr>
<tr>
<td>300</td>
<td>1.480</td>
<td>1.480</td>
<td>1.466</td>
<td>0.972</td>
</tr>
<tr>
<td>400</td>
<td>1.712</td>
<td>1.712</td>
<td>1.694</td>
<td>1.096</td>
</tr>
<tr>
<td>500</td>
<td>1.904</td>
<td>1.904</td>
<td>1.883</td>
<td>1.200</td>
</tr>
</tbody>
</table>

Fig 8. Load- deflection curves for simply supported piezoelectric shell for different values of $e_{31}$
6.1 EFFECT OF GEOMETRICAL PARAMETER $\mu$

Variation in the values of $\mu$ is shown in Table 7 and Fig 9. It shows that with increase in the value of $\mu$ from 20 to 50, $w_o/h$ decreases by 15.26% at $Q=500$, by 36.57% at $Q=100$, and by 51.6% at $Q=150$.

Table 7. Load-deflection curves for simply supported piezoelectric shell for different values of $\mu$.

![Graph showing load-deflection curves for different values of $\mu$.]

7 EFFECT OF PIEZOELECTRIC PROPERTIES.

Different values of displacement are plotted against $Q$ for piezoelectric shell as well as for the shell without piezoelectric properties for the constant $E_{31}=-171 \times 10^{-12}$. As is evident from Table 8 and Fig. 10, there is a decrease in the value of $w_o/h$ with the application of piezoelectric properties to the shell. The decrease is about 0.88% at $Q=500$.

Table 8. Load-deflection curves for simply supported shell with and without piezoelectric properties.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_o/h$ for orthotropic shell</th>
<th>$w_o/h$ for piezoelectric shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.518</td>
<td>0.517</td>
</tr>
<tr>
<td>200</td>
<td>0.917</td>
<td>0.912</td>
</tr>
<tr>
<td>300</td>
<td>1.220</td>
<td>1.212</td>
</tr>
<tr>
<td>400</td>
<td>1.463</td>
<td>1.451</td>
</tr>
<tr>
<td>500</td>
<td>1.666</td>
<td>1.651</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_o/h$ for $\mu=20$</th>
<th>$w_o/h$ for $\mu=50$</th>
<th>$w_o/h$ for $\mu=100$</th>
<th>$w_o/h$ for $\mu=150$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.831</td>
<td>0.518</td>
<td>0.297</td>
<td>0.205</td>
</tr>
<tr>
<td>200</td>
<td>1.260</td>
<td>0.917</td>
<td>0.574</td>
<td>0.406</td>
</tr>
<tr>
<td>300</td>
<td>1.552</td>
<td>1.220</td>
<td>0.825</td>
<td>0.598</td>
</tr>
<tr>
<td>400</td>
<td>1.778</td>
<td>1.463</td>
<td>1.048</td>
<td>0.780</td>
</tr>
<tr>
<td>500</td>
<td>1.966</td>
<td>1.666</td>
<td>1.247</td>
<td>0.951</td>
</tr>
</tbody>
</table>
Fig 10. Load-deflection curves for simply supported shell with and without piezoelectric properties.

8 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS.

Variation in the value of piezoelectric constants $e_{31}$ is shown in Table 9 and Fig 11. By taking $e_{31} = -171 \times 10^{-12}$, there is negligible decrease in the value of $w_0/h$. But if this value is taken as $e_{31} = -171 \times 10^{-4}$ there is an unexpected decrease in it. It is about 32.96% at $Q=500$. Thus, it can be inferred that the effect of increase in the value of piezoelectric constant is to reduce the value of $w_0/h$.

Table 9. Load-deflection variations for simply supported piezoelectric shell for different values of $e_{31}$.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-12}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-4}$</th>
<th>$w_0/h$ for $e_{31} = -171 \times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.518</td>
<td>0.518</td>
<td>0.451</td>
</tr>
<tr>
<td>200</td>
<td>0.917</td>
<td>0.917</td>
<td>0.703</td>
</tr>
<tr>
<td>300</td>
<td>1.220</td>
<td>1.220</td>
<td>0.875</td>
</tr>
<tr>
<td>400</td>
<td>1.463</td>
<td>1.463</td>
<td>1.007</td>
</tr>
<tr>
<td>500</td>
<td>1.666</td>
<td>1.666</td>
<td>1.117</td>
</tr>
</tbody>
</table>

Fig 11. Load-deflection curves for simply supported piezoelectric shell for different values of $e_{31}$.

9 NON-DIMENSIONAL LOAD VERSUS ELECTRIC POTENTIAL CHARACTERISTICS

9.1 EFFECT OF VARIATION IN $\eta$.

The effect of variation in $\eta$ is shown in Fig 12, and Table 10 clearly depicts that there is a decreasing trend in the value of $\phi$ with increase in the value of $\eta$. For non-dimensional load $Q = 500$, the decrease is 25.37%.
when $\eta$ is increased from 0.25 to 1.25, a decrease of 55.58% when $\eta$ is increased to 2.5. Thus it can be asserted that with increase in $\eta$, the value of electric potential $\Phi_0$ decreases.

Table 10. Electric potential non-dimensional load variations with different values of $\eta$

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$\Phi_0 \times 10^6$ for $\eta = 0.25$</th>
<th>$\Phi_0 \times 10^6$ for $\eta = 1.25$</th>
<th>$\Phi_0 \times 10^6$ for $\eta = 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>1.051</td>
<td>0.550</td>
<td>0.096</td>
</tr>
<tr>
<td>200</td>
<td>2.461</td>
<td>1.564</td>
<td>0.621</td>
</tr>
<tr>
<td>300</td>
<td>3.758</td>
<td>2.584</td>
<td>1.288</td>
</tr>
<tr>
<td>400</td>
<td>4.954</td>
<td>3.566</td>
<td>1.989</td>
</tr>
<tr>
<td>500</td>
<td>6.072</td>
<td>4.507</td>
<td>2.697</td>
</tr>
</tbody>
</table>

![Fig 12. Electric potential non-dimensional load characteristic](image)

10 EFFECT OF GEOMETRICAL PARAMETER $\lambda$.

The effect of variation in the values of $\lambda$ on $\Phi_0$ is shown in table 11 and Fig.13. With increase in the value of $\lambda$ from 20 to 50, the value of $\Phi_0$ increases by 6.35% at $Q = 500$, by 16.3% at $Q=100$, and by 25.41% at $Q=150$.

Table 11. Electric potential versus non-dimensional load variations for different values of $\lambda$

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$\Phi_0$ at $\lambda = 20$</th>
<th>$\Phi_0$ at $\lambda = 50$</th>
<th>$\Phi_0$ at $\lambda = 100$</th>
<th>$\Phi_0$ at $\lambda = 150$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>1.051</td>
<td>0.851</td>
<td>0.605</td>
<td>0.440</td>
</tr>
<tr>
<td>200</td>
<td>2.461</td>
<td>2.161</td>
<td>1.734</td>
<td>1.390</td>
</tr>
<tr>
<td>300</td>
<td>3.758</td>
<td>3.414</td>
<td>2.897</td>
<td>2.450</td>
</tr>
<tr>
<td>400</td>
<td>4.954</td>
<td>4.585</td>
<td>4.015</td>
<td>3.505</td>
</tr>
<tr>
<td>500</td>
<td>6.072</td>
<td>5.686</td>
<td>5.082</td>
<td>4.529</td>
</tr>
</tbody>
</table>
EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS

The effects of variation in piezoelectric constants $e_{31}$ are shown in Table 12 and Fig 14. With $e_{31} = -171 \times 10^{-12}$, there is negligible increase in the value of $\Phi_0$ corresponding to $Q$. It is about $5.68 \times 10^{-6}$ at $Q = 500$. When the constant is increased to $e_{31} = -171 \times 10^{-8}$, the value of $\Phi_0$ increases by 82.82% at $Q = 500$. The increase in the value of $e_{31} = -171 \times 10^{-4}$ shows an increase of about $10^7$%. And if this value is increased to $e_{31} = -171 \times 10^{-4}$, there is an unexpected increase in the value of $\Phi_0$. It is about $3.19 \times 10^7$% at $Q = 500$. Thus, it can be inferred that the effect of increase in the value of piezoelectric constant is to increase the value of $\Phi_0$.

Table 12. Electric potential – non-dimensional load variations for different values of piezoelectric constants

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$\Phi_0 \times 10^4$ at $e_{31} = -171 \times 10^{-12}$</th>
<th>$\Phi_0$ at $e_{31} = -171 \times 10^{-8}$</th>
<th>$\Phi_0$ at $e_{31} = -171 \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.851</td>
<td>0.008</td>
<td>48.409</td>
</tr>
<tr>
<td>200</td>
<td>2.161</td>
<td>0.021</td>
<td>99.934</td>
</tr>
<tr>
<td>300</td>
<td>3.414</td>
<td>0.034</td>
<td>144.78</td>
</tr>
<tr>
<td>400</td>
<td>4.585</td>
<td>0.045</td>
<td>185.3</td>
</tr>
<tr>
<td>500</td>
<td>5.686</td>
<td>0.056</td>
<td>222.766</td>
</tr>
</tbody>
</table>

Fig 13. Electric potential non-dimensional load curve for different $\lambda$.

Fig 14. Logarithmic electric potential versus non-dimensional load curves for different values of piezoelectric constants.
11.1 EFFECT OF GEOMETRICAL PARAMETER $\mu$.

The variation in the values of $\mu$ shows the same trend towards change in the values of $\phi_0$ as shown by $\eta$ and $\lambda$. With an increase in the value of $\mu$ from 20 to 50, the $\phi_0$ increases by 28.49% at $Q=500$, by 60.31% at $Q=100$, and by 77.15% at $Q=150$. The trend is shown in Table 12 and Fig 14.

Table 13. Electric potential – non-dimensional load variations for different values of $\mu$

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$\phi_0 \times 10^6$ for $\mu = 20$</th>
<th>$\phi_0 \times 10^6$ for $\mu = 50$</th>
<th>$\phi_0 \times 10^6$ for $\mu = 100$</th>
<th>$\phi_0 \times 10^6$ for $\mu = 150$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>1.051</td>
<td>0.394</td>
<td>0.120</td>
<td>0.053</td>
</tr>
<tr>
<td>200</td>
<td>2.461</td>
<td>1.285</td>
<td>0.489</td>
<td>0.236</td>
</tr>
<tr>
<td>300</td>
<td>3.758</td>
<td>2.305</td>
<td>1.036</td>
<td>0.532</td>
</tr>
<tr>
<td>400</td>
<td>4.954</td>
<td>3.335</td>
<td>1.691</td>
<td>0.923</td>
</tr>
<tr>
<td>500</td>
<td>6.072</td>
<td>4.341</td>
<td>2.410</td>
<td>1.387</td>
</tr>
</tbody>
</table>

Fig 15. Electric potential versus non-dimensional load curves for different values of $\mu$

CONCLUSIONS

Based on the results obtained, the following conclusions can be drawn.

(1). Geometric parameter $\eta$ has an adverse effect on the displacement of the shell. It decreases with increase in the value of $\eta$. It is about 9.02% when $\eta$ increases from 0.25 to 1.25, and 21.94% when $\eta$ increased to 2.5. Thus it can also be inferred that with increase in the ratio of $a/h$ or $a/R$, the deflection decreases.

(2). Geometric parameter/foundation parameter $\lambda$ also has the same effect as $\eta$. When $\lambda$ increases from 20 to 50, it decreases by 3.19%, and when $\lambda$ becomes 100, it decreases by 13.48%. Thus with increase in spring constant of the foundation, the deflection decreases.

(3). Effect of $\mu$ can be summarized as follow. It is 15.26% decrease for increase of $\mu$ from 20 to 50, 36.57% decrease for $\mu = 100$, and 51.60% decrease for $\mu = 150$. Thus it can be said that an increase in the value of shear modulus of the foundation layer adversely affects the value of deflection.

(4). Increase in the value of geometric parameter $\eta$ has positive effect on the electric potential. It increases by 25.77% for an increase in the value of $\eta$ from 0.25 to 1.25, and 55.58% for increase in $\eta$ from 0.25 to 1.25.
(5). Increase in the value of geometric parameter/foundation parameter $\lambda$ also has a positive effect on electric potential. It increases by 6.35 % for an increase in $\lambda$ from 20 to 50, 16.3 % for increase in $\lambda$ from 20 to 100, and 25.41 % for $\lambda$ increasing to 150.

(6). Increase in the value of geometric parameter $\mu$ has same effect on electric potential as $\eta$ and $\lambda$. It increases by 28.49 % for increase in $\mu$ from 20 to 50, by 60.31 % for increase in $\mu$ from 20 to 100, and 77.15 % for $\mu$ increasing to 150.

(7). Increase in the values of piezoelectric constants decreases the value of deflection and increases electric potential.

NOMENCLATURE

$A_{ij}$ Dimensions of cylindrical shell

$\alpha, \beta$ Dimensions of cylindrical shell

$[C]$ Elastic constant matrix

$c_{ij}$ Elastic constants

$D_1, D_2, D_3$ Flexural rigidities of the orthotropic shell

$\begin{align*}
\frac{E\nu^3}{12(1-\nu^2)} and \frac{E_2\nu^3}{12(1-\nu^2)}
\end{align*}$

respectively. $\{D\}$ Electrical displacement vector

$D_x, D_y, D_z$ Electrical displacement in x, y and z directions, respectively

$E_1, E_2$ Young’s Modulii of elasticity in orthogonal directions

$\{E\}$ Electric field vector

$E^\alpha$ Applied uniform electric field in radial direction

$[e]$ Piezoelectric stress coefficient matrix

$e_{ij}$ Piezoelectric stress coefficients

$e_1$ First invariant of middle surface strains

$e_2$ Second invariant of middle surface strains

$F$ Integrand

$f$ Sag, the highest elevation above the base

$G$ Shear modulus of the foundation

$H$ Thickness of the orthotropic shell

$k$ Foundation Modulus

$M$ Moments per unit length

$N$ Normal forces per unit length

$[\kappa]$ Dielectric matrix, i.e. $\kappa_{11}, \kappa_{22}, \text{ and } \kappa_{33}$
\begin{align*}
\text{\texttt{\textcolor{black}{l_{\text{min}}}}: Minimum dimension of the shell in the plan} \\
\text{\texttt{\textcolor{black}{Q}: Non-dimensional load}} \\
\text{\texttt{\textcolor{black}{q}: Uniform normal pressure on the concave side of the shell}} \\
\text{\texttt{\textcolor{black}{R}: Radius of curvature of cylindrical shell}} \\
\text{\texttt{\textcolor{black}{U}: Potential energy}} \\
\text{\texttt{\textcolor{black}{u, v, w}: Displacement components in r, \theta and z directions respectively}} \\
\text{\texttt{\textcolor{black}{w_o}: Maximum deflection at the center of the shell}} \\
\text{\texttt{\textcolor{black}{r, \theta, z}: Cylindrical coordinates}} \\
\text{\texttt{\textcolor{black}{x, y, z}: Rectangular coordinates}} \\
\text{\texttt{\textcolor{black}{z}: Distance of the middle surface of the shell from the base plane}} \\
\text{\texttt{\textcolor{black}{{\varepsilon}_{ij}: Strains}}} \\
\text{\texttt{\textcolor{black}{\gamma_{xy}, \gamma_{yz}, \gamma_{zx}: Shear strains in xy, yz and zx planes, respectively}}} \\
\text{\texttt{\textcolor{black}{\{\varepsilon\}: Strain vector}}} \\
\text{\texttt{\textcolor{black}{\eta}: Geometric parameter}} \\
\text{\texttt{\textcolor{black}{\lambda}: Non-dimensional foundation modulus}} \\
\text{\texttt{\textcolor{black}{\mu}: Non-dimensional (foundation) shear modulus, Ga^2/D_{11}}} \\
\text{\texttt{\textcolor{black}{\nu_1 \& \nu_2: Poisson's ratio in orthogonal directions.}}} \\
\text{\texttt{\textcolor{black}{\sqrt{\nu_1 \times \nu_2}:}}} \\
\text{\texttt{\textcolor{black}{\sigma_q: Stress}}} \\
\text{\texttt{\textcolor{black}{\{\sigma\}: Stress vector}}} \\
\text{\texttt{\textcolor{black}{\phi_o: Increment of the electric potential from an initial State.}}} \\
\end{align*}

\textbf{REFERENCES}


