CLASSES OF CHARGED ANISOTROPIC STARS WITH POLYTROPIC EQUATION OF STATE

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ABSTRACT
In this paper, we study the Einstein-Maxwell equations with anisotropic pressures considering a polytropic equation of state in presence of an electric field and a modified version of metric potential proposed by Korkina-Orlyanskii (1991). We obtain two new classes of exact models that satisfy all physical features expected in a realistic star. Graphical analysis indicates that physical properties as energy density, radial pressure, charge density and anisotropy profiles and the measures of anisotropy are consistent with seminal treatments which suggest great importance in the description of relativistic compact objects.

Keywords: *Einstein-Maxwell equations, Anisotropic pressures, Polytropic equation of state, Measures of anisotropy, Relativistic compact objects*.

1. INTRODUCTION

The modeling of super-dense charged matter configurations in strong gravitational fields has received much attention in the last decades because of its relevance in relativistic astrophysics [1,2]. These models allow us to explain the behavior of massive objects as neutron stars, quasars, pulsars, black holes and white dwarfs [3] and require the finding of exact solutions of the Einstein-Maxwell system [4]. The family obtained from solutions should be studied in order to determine its physical characteristics. In this framework, Delgaty and Lake [4] discussed some relevance physical requirements and constructed several analytical solutions that describe static perfect fluid.

It is important to mention the contributions of Schwarzschild [5], Tolman [6], Oppenheimer and Volkoff [7] and Chandrasekhar [8] in the development of the first theoretical models of stellar objects. Schwarzschild [5] found analytical solutions that allowed describing a star with a uniform density, Tolman [6] developed a method to find solutions of static spheres of fluid and Oppenheimer and Volkoff [7] used Tolman's solutions to study the gravitational balance of neutron stars and Chandrasekhar [8] generated some models of white dwarfs in presence of relativistic effects. The works of Zwicky [9] are also very relevant in the description and the identification of astronomical objects known as supernovae.

A considerable number of exact models with electric charge from the Einstein-Maxwell field equations have been generated by Gupta and Maurya [10], Kiess [11], Mafa Takisa and Maharaj [12], Malaver and Kasmaei [13], Malaver [14,15], Ivanov [16] and Sunzu et al [17]. The presence of charge produces values for the redshift, luminosity and maximum mass for stars which are different for neutral matter [12, 18].

In theoretical works of realistic stellar models, it is important to include the pressure anisotropy [19-30]. Bowers and Liang [31] extensively discuss the effect of pressure anisotropy in general relativity. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [32] and other physical phenomena as well as the presence of an electrical field [33].

Many researchers have used various analytical techniques to try in order to obtain solutions of Einstein field equations for relativistic stars as it has been shown by Komathiraj and Maharaj [34], Thirukkanesh and Maharaj [35], Maharaj et al. [36], Thirukkanesh and Ragel [32,33], Feroze and Siddiqui [34,35], Sunzu et al. [36], Pant et al. [37] and Malaver [38-41]. These studies suggest that the Einstein-Maxwell field equations are very important in the description of the stellar structures.

The theoretical modelling with various equations of state has allowed the study and detailed description of a wide variety of ultra-compact objects as strange and neutron stars. Komathiraj and Maharaj [42], Malaver [43], Bombaci [44], Thirukkanesh and Maharaj [35], Dey et al. [45] and Usov [33] considered linear equations of state for quark stars. The models of Feroze and Siddiqui [46] and Maharaj and Mafa Takisa [47] with quadratic equation of state for the matter distribution are very important in the brane world treatment and the study of dark energy. Bhar and

Murad [48] obtained new relativistic stellar models with a particular type of metric function and a generalized Chaplygin equation of state. Also recently Tello-Ortiz et al. [49] have found anisotropic fluid sphere solution with a modified generalized Chaplygin equation of state. Mafa Takisa and Maharaj [12] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state specifying particular forms for the gravitational potential and electric field intensity. Thirukkanesh and Ragel [50] obtained uncharged models of anisotropic fluids with polytropic equation of state consistent with experimental observations. Malaver [39] generated new exact solutions considering Van der Waals modified equation of state with polytropic exponent. More recently, Singh et al. [51] presented new models for anisotropic stars by taking a modified polytropic equation in a Korkina-Orlyanskii space-time [52] and analysed four different models for distinct polytropic index.

The polytropic equation of state was successfully used in describing the physical properties of some stellar objects [53]. Nilsson and Uggla [54] solved the gravitational field equations for static spherically symmetric perfect fluid models with the equation of state $p_r = \alpha\rho^{1+1/\eta}$ and obtained two important formulations for compact objects with different polytropic indexes η. Arbañil and Moraes [55] analyzed the effect of the cosmological constant in the stellar equilibrium configuration and radial stability of relativistic polytropic spheres. Abellán et al. [56] developed an algorithm to construct static spherically symmetric anisotropic solutions for general relativistic polytropes.

Many analytical difficulties arise when polytropic treatment is included. Because pressure dependence with energy density does not allow field equations to be integrated [50]. In this paper, we obtained new models for charged anisotropic matter considering a polytropic equation of state and a modified form of gravitational potential proposed by Korkina-Orlyanskii [52]. The article is organized as follows: In section 2, we present Einstein´s field equations for anisotropic fluid distribution. In section 3,we make a particular choice for gravitational potential $Z(x)$ and the electric field intensity that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, physical acceptability conditions are discussed. The physical properties and physical validity of these new solutions are analyzed in section 5. The conclusions of the results obtained by computational implementations are shown in the section 6.

2. EINSTEIN-MAXWELL EQUATIONS

We consider a spherically symmetric, static and homogeneous space-time. In Schwarzschild coordinates, the metric is given by:

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by [35]:

$$\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda} = \rho+\frac{1}{2}E^2 \tag{2}$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} = p_r-\frac{1}{2}E^2 \tag{3}$$

$$e^{-2\lambda}\left(\nu''+\nu'^2+\frac{\nu'}{r}-\nu'\lambda'-\frac{\lambda'}{r}\right) = p_t+\frac{1}{2}E^2 \tag{4}$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}(r^2E)' \tag{5}$$

Where ρ is the energy density, p_r is the radial pressure, E is electric field intensity, is p_t the tangential pressure and primes denote differentiations with respect to r. Using the transformations $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A_*^2 y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0 suggested by Durgapal and Bannerji [57], the Einstein field equations can be written as:

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c}+\frac{E^2}{2c} \tag{6}$$

$$4Z\frac{\dot{y}}{y}-\frac{1-Z}{x}=\frac{p_r}{c}-\frac{E^2}{2c} \tag{7}$$

$$4xZ\frac{\ddot{y}}{y}+(4Z+2x\dot{Z})\frac{\dot{y}}{y}+\dot{Z}=\frac{p_t}{c}+\frac{E^2}{2c} \tag{8}$$

$$p_t=p_r+\Delta \tag{9}$$

$$\frac{\Delta}{c}=4xZ\frac{\ddot{y}}{y}+\dot{Z}\left(1+2x\frac{\dot{y}}{y}\right)+\frac{1-Z}{x}-\frac{E^2}{c} \tag{10}$$

$$\sigma^2=\frac{4cZ}{x}\left(x\dot{E}+E\right)^2 \tag{11}$$

σ is the charge density, $\Delta = p_t - p_r$ is the is the anisotropy factor and dots denote differentiations with respect to x.With the transformations of [57], the mass within a radius r of the sphere takes the form:

$$M(x)=\frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\,\rho(x)dx \tag{12}$$

In this paper, we assume a polytropic equation of state relating the radial pressure to the energy density given by:

$$p_r=\alpha\rho^{\Gamma} \tag{13}$$

where $\Gamma = 1+ 1/\eta$, η is the polytropic index and α is an arbitrary constant.

3. THE NEW MODELS

In this research, we solve the Einstein-Maxwell field equations choosing specific forms for the gravitational potential *Z(x)* and the electric field intensity *E*. Following Singh et al. [51] for *Z(x)*, we make the choice

$$Z(x)=\frac{ax+1}{2ax+1} \tag{14}$$

where a is a real constant. This potential is regular at the stellar center and well behaved in the interior of the sphere.

For the electric field, we make the choice

$$\frac{E^2}{2c}=\frac{kax}{\left(1+2ax\right)^2} \tag{15}$$

which is finite at the origin and remains regular and continuous in the stellar interior. *K* is a constant and positive real value. The equation. (15) produces charged anisotropic models with a polytropic equation of state.Using (14) and (15) in equation. (6), we obtain for the energy density as follows:

$$\frac{\rho}{c}=\frac{(2a-k)ax+3a}{\left(1+2ax\right)^2} \tag{16}$$

Substituting equation. (16) in equation. (12), the expression of the mass function can be written as

$$M(x)=\frac{\left[4\left(2a^2-ak\right)x-3k\right]\sqrt{x}}{16a\sqrt{c}\left(1+2ax\right)}+\frac{3\sqrt{2}\arctan\left(\sqrt{2ax}\right)}{32a\sqrt{ac}} \tag{17}$$

Replacing *Z(x)* and *E* in equation (11), the charge density must have the form

$$\sigma^2=\frac{2akc^2\left(ax+1\right)\left(3+2ax\right)^2}{\left(1+2ax\right)^5} \tag{18}$$

In this paper, we considered two cases of polytropic index η=1/2, 1 when anisotropy and the electric field are present.

For the case η=1/2, substituting equation. (16) in equation. (13), the radial pressure can be written in the form

$$p_r=\frac{\alpha c^3\left[\left(2a-k\right)ax+3a\right]^3}{\left(1+2ax\right)^6} \tag{19}$$

With (19), (15) and (14) in equation (7), we have

$$\frac{\dot{y}}{y}=\frac{\alpha c^2\left[\left(2a-k\right)ax+3a\right]^3}{4\left(1+2ax\right)^5\left(1+ax\right)}-\frac{kax}{4\left(1+ax\right)\left(1+3ax\right)}+\frac{a}{4\left(1+ax\right)} \tag{20}$$

Integrating equation (20), we obtain

$$y(x)=c_1\left(ax+1\right)^{A*}\left(2ax+1\right)^B e^{\frac{\alpha c^2\left(Cx^3+Dx^2+Ex+F\right)}{384a\left(2ax+1\right)^4}} \tag{21}$$

where for the convenience we have let

$$A^*=-\frac{a^3\alpha c^2+3a^2\alpha c^2k+3a\alpha c^2k^2+\alpha c^2k^3-a+k}{4a} \tag{22}$$

$$B=\frac{2a^3\alpha c^2+6a^2\alpha c^2k+6a\alpha c^2k^2+2\alpha c^2k^3+k}{8a} \tag{23}$$

$$C=768a^6+2304a^5k+2304a^4k^2+768a^3k^3 \tag{24}$$

$$D=768a^5+3168a^4k+2736a^3k^2+984a^2k^3 \tag{25}$$

$$E=-64a^4+1824a^3k+1248a^2k^2+440ak^3 \tag{26}$$

$$F=-320a^3+264a^2k+192ak^2+67k^3 \tag{27}$$

and c_1 is the constant of integration.

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as:

$$e^{2\lambda}=\frac{1+2ax}{1+ax} \tag{28}$$

$$e^{2\nu}=c_1^2A^2\left(ax+1\right)^{2A*}\left(2ax+1\right)^{2B}e^{\frac{\alpha c^2\left(Cx^3+Dx^2+Ex+F\right)}{192a\left(2ax+1\right)^4}} \tag{29}$$

and for the measure of anisotropy Δ ,we have:

$$\Delta=\frac{4xc\left(ax+1\right)}{\left(2ax+1\right)}\left[\begin{array}{l}\frac{\left(A*^2-A*\right)a^2}{\left(ax+1\right)^2}+\frac{4Aa^2B}{\left(ax+1\right)\left(2ax+1\right)}+\frac{2aA\alpha C^2}{\left(ax-1\right)}\left(\frac{3Cx^2+2Dx+E}{384a\left(2ax+1\right)^4}-\frac{Cx^3+Dx^2+Ex+F}{48\left(2ax+1\right)^5}\right)\\+\frac{4\left(B^2-B\right)a^2}{\left(2ax+1\right)^2}+\frac{4aB\alpha C^2}{\left(2ax+1\right)}\left(\frac{3Cx^2+2Dx+E}{384a\left(2ax+1\right)^4}-\frac{Cx^3+Dx^2+Ex+F}{48\left(2ax+1\right)^5}\right)\\+\frac{\alpha C^2}{\left(2ax+1\right)}\left(\frac{6Cx+D}{384a\left(2ax+1\right)^3}-\frac{3Cx^2+Dx+E}{24\left(2ax+1\right)^4}+\frac{5a\left(Cx^3+Dx^2+Ex+F\right)}{24\left(2ax+1\right)^5}\right)\\+\left(\frac{\alpha C^2\left(3Cx^2+2Dx+E\right)}{384a\left(2ax+1\right)^4}-\frac{\alpha C^2\left(Cx^3+Dx^2+Ex+F\right)}{48\left(2ax+1\right)^5}\right)^2\end{array}\right]$$
$$-\frac{a}{\left(2ax+1\right)^2}\left[1+2x\left(\frac{aA*}{ax+1}+\frac{2Ba}{2ax+1}+\frac{\alpha C^2\left(3Cx^2+2Dx+E\right)}{384a\left(2ax+1\right)^4}-\frac{\alpha C^2\left(Cx^3+Dx^2+Ex+F\right)}{48\left(2ax+1\right)^5}\right)\right]$$
$$+\frac{a}{2ax+1}-\frac{2kax}{\left(2ax+1\right)^2} \tag{30}$$

With η=1,we obtain for the radial pressure

$$p_r=\frac{\alpha c^2\left[\left(2a-k\right)ax+3a\right]^2}{\left(1+2ax\right)^4} \tag{31}$$

and the equation. (7) can be written as:

$$\frac{\dot{y}}{y}=\frac{\alpha c\left[\left(2a-k\right)ax+3a\right]^2}{4\left(1+2ax\right)^3\left(1+ax\right)}-\frac{kax}{4\left(1+ax\right)\left(1+3ax\right)}+\frac{a}{4\left(1+ax\right)} \tag{32}$$

Integrating (32), we can obtain

$$y(x)=c_2\left(ax+1\right)^G\left(2ax+1\right)^He^{\frac{Ix+J}{32a\left(2ax+1\right)^2}} \tag{33}$$

Again for convenience we have let

$$G = -\frac{a^2\alpha c + 2a\alpha ck + \alpha ck^2 - a + k}{4a} \tag{34}$$

$$H = \frac{2a^2\alpha c + 4a\alpha ck + 2\alpha ck^2 + k}{8a} \tag{35}$$

$$I = 48a^2\alpha ck + 12a\alpha ck^2 \tag{36}$$

$$J = 16a\alpha ck - 16a^2\alpha c + 5\alpha ck^2 \tag{37}$$

where c_2 is the constant of integration.

For the metric functions $e^{2\lambda}$ and $e^{2\nu}$ we have

$$e^{2\lambda} = \frac{1+2ax}{1+ax} \tag{38}$$

$$e^{2\nu} = c_2^2 A^2 (ax+1)^{2G} (2ax+1)^{2H} e^{\frac{Ix+J}{16a(2ax+1)^2}} \tag{39}$$

and for the anisotropy Δ, we obtain

$$\Delta = \frac{4xc(ax+1)}{(2ax+1)}\left[\begin{array}{l} \frac{(G^2-G)a^2}{(ax+1)^2} + \frac{Ga^2}{(ax+1)(2ax+1)} + \frac{2Ga}{(ax+1)}\left(\frac{I}{32a(2ax+1)^2} - \frac{Ix+J}{8(2ax+1)^3}\right) \\ + \frac{4(H^2-H)a^2}{(2ax+1)^2} + \frac{4Ha}{(2ax+1)}\left(\frac{I}{32a(2ax+1)^2} - \frac{Ix+J}{8(2ax+1)^3}\right) \\ - \frac{I}{4(2ax+1)^3} + \frac{3a(Ix+J)}{4(2ax+1)^4} + \left(\frac{I}{32a(2ax+1)^2} - \frac{Ix+J}{8(2ax+1)^3}\right)^2 \end{array}\right]$$
$$- \frac{a}{(2ax+1)^2}\left[1 + 2x\left(\frac{2Ga}{ax+1} + \frac{2Ha}{2ax+1} + \frac{I}{32a(2ax+1)^2} - \frac{Ix+J}{8(2ax+1)^3}\right)\right]$$
$$+ \frac{a}{2ax+1} - \frac{2kax}{(2ax+1)^2} \tag{40}$$

4.REQUIREMENTS OF PHYSICAL ACCEPTABILITY

A physically acceptable interior solution of the field equations must satisfy the following physical conditions [4,48,50]:

(i) Regularity of the metric potentials in the stellar interior and at the origin.
(ii) The radial pressure should be positive and a decreasing function of radial coordinate for $0 \leq r \leq R$.
(iii) The energy density should be well defined, positive and a decreasing function of the radial parameter.
(iv) $p_r > 0$ and $\rho > 0$ in the origin.

(v) Any physically acceptable solution must satisfy the causality condition where the radial speed of sound should be less than speed of light throughout the model, that is

$$0 \le \frac{dp_r}{d\rho} \le 1$$

(vi) For the anisotropic case, the radial and the tangential pressure are equal to zero at the centre r=0, i.e. Δ(r=0) =0.
(vii) In the surface of the sphere, it should be matched with the Reissner–Nordström exterior solution, for which the metric is given by:

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2 d\varphi^2\right) \tag{41}$$

through the boundary r=R where M is the total mass of the star.

The conditions (ii), (iii) and (iv) imply that radial pressure and energy density should reach a maximum at the centre and decreasing towards the surface of the sphere [48].

5. SOME PHYSICAL FEATURES OF THE NEW MODELS

Now, we show the physical analysis for the proposed new models:

For the case η=1/2, the metric potentials $e^{2\lambda}$ and $e^{2\nu}$ should remain positive throughout the stellar interior and in the origin $e^{2\lambda(0)} = 1$, $e^{2\nu(0)} = C_1^2 A^2 e^{\frac{\alpha c^2 F}{192a}}$. We show at *r=0* , $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ and this makes it possible to verify that the gravitational potentials are regular at the centre *r=0*.

The energy density and radial pressure are positive and well behaved inside the sphere. In the centre *r=0* , $\rho(0) = 3ac$, $p_r(0) = 27\alpha c^3 a^3$ and both are positive if *a, c > 0* .

Now, we In the surface of the star *r=R* , we have $p_r(r = R) = 0$ and $R = \sqrt{\frac{3}{c(k-2a)}}$.

For the pressure and density gradients inside the stellar interior, we can obtain respectively

$$\frac{d\rho}{dr} = \frac{2ac^2 r\left[(2a-k)\left(1+2acr^2\right)-4\right]}{\left(1+2acr^2\right)^3} < 0 \tag{42}$$

$$\frac{dp_r}{dr} = \frac{6a\alpha c^4 r\left[(2a-k)acr^2+3a\right]^2\left[(2a-k)\left(1+2acr^2\right)-4\left((2a-k)acr^2+3a\right)\right]}{\left(1+2acr^2\right)^7} < 0 \tag{43}$$

Then the energy density and radial pressure decrease from the centre to the surface of the star if $a < k$.

With the equation. (17) and the transformations of Durgapal and Bannerji [57], we have for the total mass of the star

$$M(r=R) = \frac{3}{16a\sqrt{c}}\left[\frac{\arctan\left(\sqrt{\frac{6a}{k-2a}}\right)}{\sqrt{2a}} - \sqrt{3(k-2a)}\right] \tag{44}$$

In order to maintain of causality, the radial sound speed defined as $v_{sr}^2 = \frac{dp_r}{d\rho}$ should be within the limit $0 \le v_{sr}^2 \le 1$ in the interior of the star. For this model

$$v_{sr}^2 = \frac{dp_r}{d\rho} = \frac{3\alpha c^2\left((2a-k)ax+3a\right)^2}{\left(1+2ax\right)^4} \tag{45}$$

and for the equation. (45), we can impose the condition

$$0 \leq \frac{3\alpha c^2\left((2a-k)acr^2+3a\right)^2}{\left(1+2acr^2\right)^4} \leq 1 \tag{46}$$

With η=1, we have $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_2^2e^{\frac{J}{16a}}$ in the origin and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$. Again, the metric potentials are regular in $r=0$. In the centre, $p_r(0)=9\alpha c^2a^2$ and is positive if *α, c, a* > 0. As the radial pressure decreases from the centre to the surface of the star, we have that for all $0<r<R$

$$\frac{dp_r}{dr}=\frac{4a\alpha c^3r\left[(2a-k)acr^2+3a\right]\left[(2a-k)\left(1+2acr^2\right)-4\left((2a-k)acr^2+3a\right)\right]}{\left(1+2acr^2\right)^5}<0 \tag{47}$$

For this case, the condition $0 \leq v_{sr}^2 \leq 1$ also implies that:

$$0 \leq \frac{2\alpha c\left((2a-k)acr^2+3a\right)}{\left(1+2acr^2\right)^2} \leq 1 \tag{48}$$

On the boundary *r=R*, the interior solution should match with the exterior Reissner–Nordström space–time

$$ds^2=-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2+r^2\left(d\theta^2+\sin^2 d\varphi^2\right)$$

and this requires that the continuity of e^{ν} and e^{λ} across the boundary *r=R* must be

$$e^{2\nu}=e^{-2\lambda}=1-\frac{2M}{R}+\frac{Q^2}{R^2} \tag{49}$$

Then for the matching conditions, we obtain:

$$\frac{2M}{R}=\frac{acR^2\left(1+2c(a+k)R^2\right)}{\left(1+2acR^2\right)^2} \tag{50}$$

The figures 1,2,3,4, and 5 demonstrate the dependence of *ρ*, *M(x)* , σ^2 , $e^{2\lambda}$, $\frac{d\rho}{dr}$ with the radial coordinate for *a=0.2*, *k= 0.446875* , *c=1* and a stellar radius of *r= 8 km* . These variables are independent of polytropic index *η* and the α parameter.

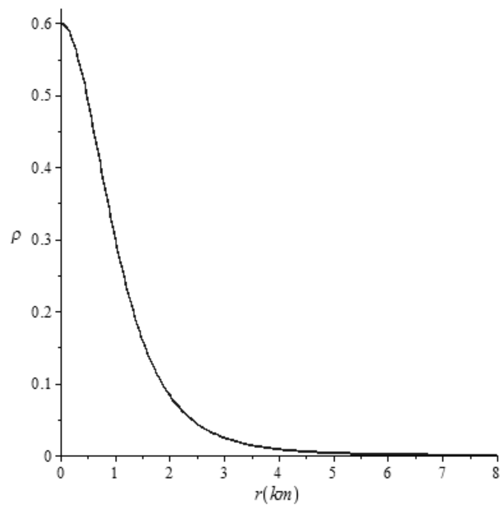

Figure 1. Variation of the energy density with radial coordinate for the two studied cases.

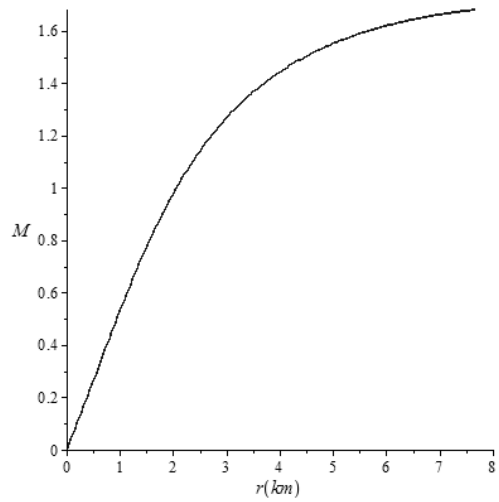

Figure 2. Variation of mass function with radial coordinate for the two studied cases.

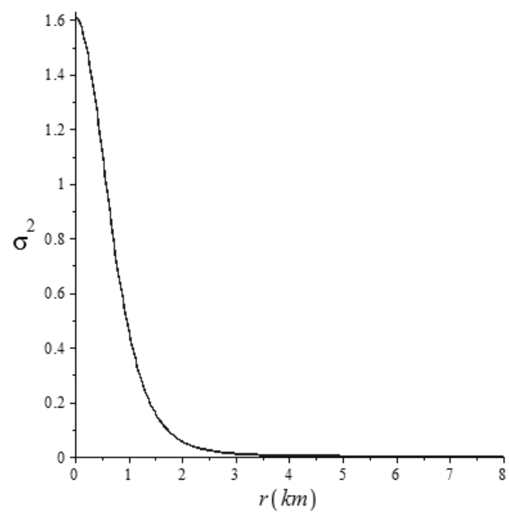

Figure 3**.** Variation of charge density with radial coordinate

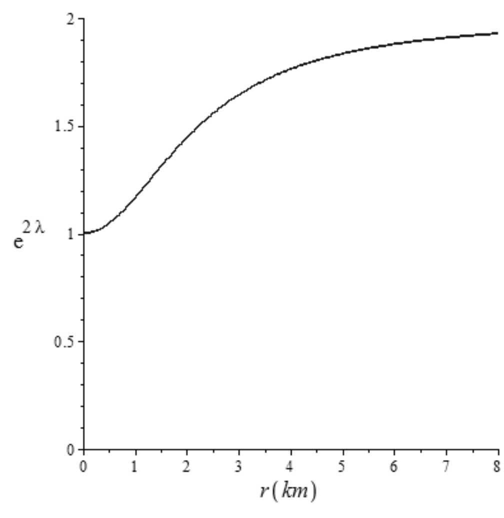

Figure 4**.** Metric potential $e^{2\lambda}$ with radial coordinate.

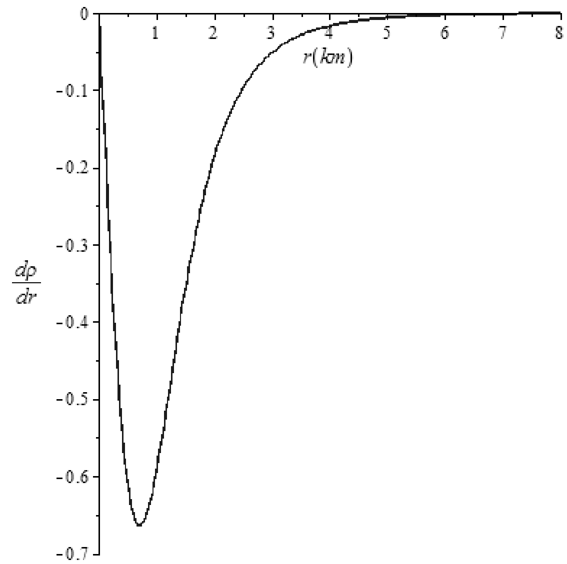

Figure 5**.** Variation of gradient of density with radial coordinate.

In the set of figures 6, 7, 8, 9 and 10 are shown the variation of p_r , v_{sr}^2 , Δ, $e^{2\nu}$, $\frac{dp_r}{dr}$ with the radial coordinate for *a=0.2*, *k= 0.446875* , *α=1/2*, *c=1* and a stellar radius of *r= 8 km*. The two studied cases of polytropic indexes were considered *η=1/2, 1*.

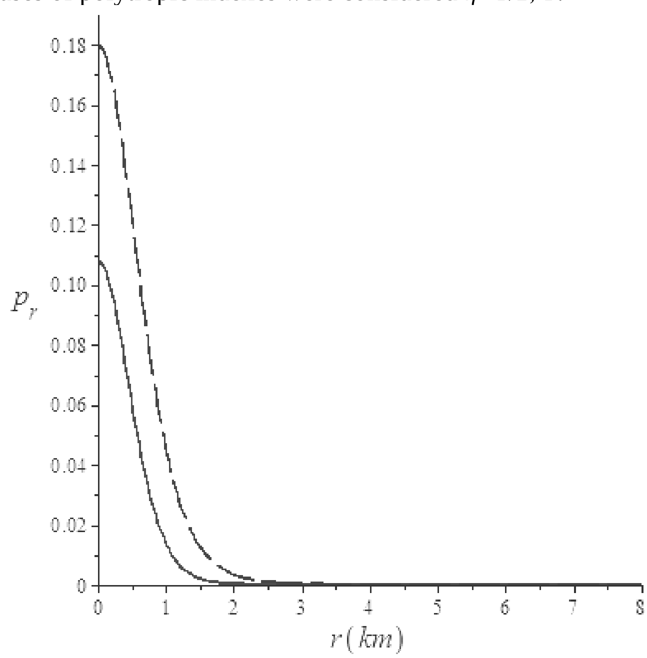

Figure 6**.** Variation of radial pressure with radial coordinate for *η=1/2* (solid line) and *η=1* (long-dash line).

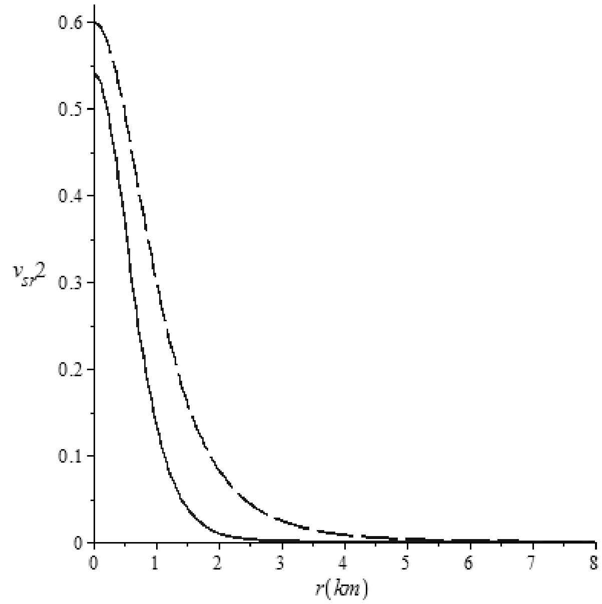

Figura 7**.** Variation of radial speed sound with radial coordinate for *η=1/2* (solid line) and *η=1* (long-dash line).

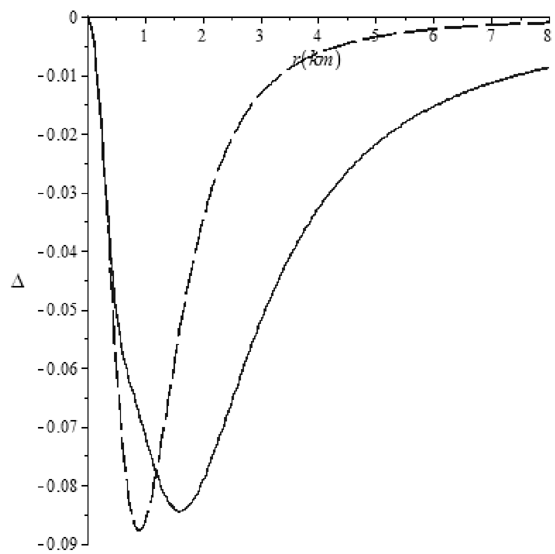

Figure 8**.** Variation of anisotropy with radial coordinate for *η=1/2* (solid line) and *η=1* (long-dash line).

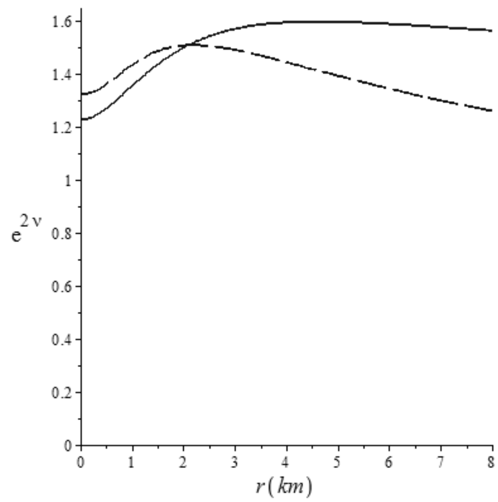

Figure 9. Variation of metric function $e^{2\nu}$ with radial coordinate for $\eta=1/2$ (solid line) and $\eta=1$ (long-dash line).

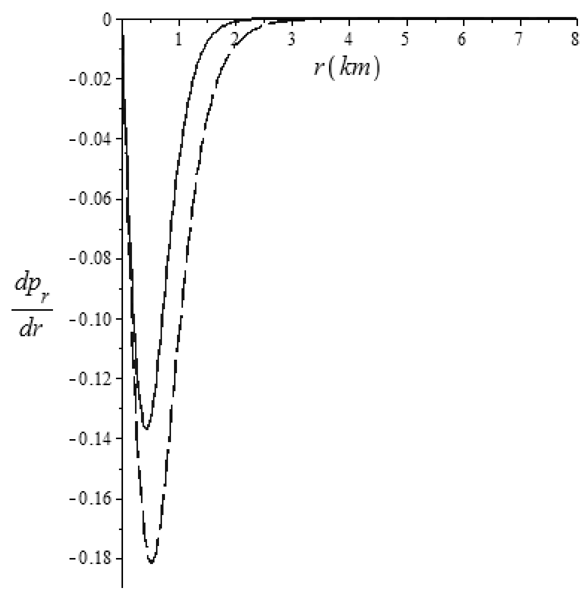

Figure 10. Variation of gradient of radial pressure with radial coordinate for η=1/2 (solid line) and η=1 (long-dash line).

In the figure 1, it is observed that the energy density is a finite, continuous and decreasing function for two studied cases. The figure 2 shows that the mass function is regular, well behaved and strictly increasing from the centre to the surface of the star. The charge density is free of singularities at the origin, non-negative and decreases with the radial parameter in figure 3. The metric potential $e^{2\lambda}$ in figure 4 is a continuously growing function inside of the sphere. In figure 5, the gradient of energy density is negative throughout the star.

In the second set of plots, physical behavior is represented for the models with polytropic indexes $\eta=1/2$ and $\eta=1$. In the figure 6, the radial pressure is a finite, decreasing and vanishes at the boundary for the two new solutions. In figure 7, the radial speed of sound $\frac{dp_r}{d\rho}$ is always less than the unity and the causality condition is maintained in the stellar interior and it is important physical requirement as indicated Delgaty and Lake [4]. The anisotropy in the figure 8 reaches a minimum in the interior of the star and then increases in the two proposed models. In figure 9, the metric function $e^{2\nu}$ is continuous, well behaved, initially increases, reaches a maximum and then decreases. In figure 10,the gradients of radial pressure $\frac{dp_r}{dr}$ also are negative for 0 ≤ r ≤ R.

As discussed above, obtained results and their comparison can be corresponded with parameter values *a=0.2* and *k=0.446875* in which they produce the mass 1.69 $M_{\odot}$ which can corresponds to astronomical object PSR J1903 + 327 results [58]. It will be said in the section of conclusions to summarize all results that were compared with each other and the more information will be mentioned.

5. CONCLUSIONS

In this paper, we have presented new models of charged anisotropic stars considering a modified version of gravitational potential proposed for Korkina-Orlyanskii space-time model and a polytropic equation of state. These models may be used in the description of compact objects with anisotropy and in presence of an electric field. A graphical analysis shows that the radial pressure, metric functions, energy density, mass function and anisotropy are regular at the origin and well behaved in the interior. The new solutions match smoothly with exterior of the Reissner–Nordström space-time at the boundary *r=R*.

The obtained models can be related to some astronomical objects such as 4U 1608 − 52, 4U 1820 – 30, Vela X − 1 and PSR J1903 + 327.Because matter variables and the gravitational potentials of this work are consistent with the physical analysis of these stars. In particular, the parameter values *a=0.2* and *k=0.446875* generate the mass 1.69$M_{\odot}$ which can corresponds to PSR J1903 + 327 results. We use the values for *a* and *k* to analyze the physical features associated with the matter, charge and gravitational potential and the graphical approach suggests that the model for the star PSR J1903 + 327 is well behaved. It is expected that the results of this study can contribute to modeling of relativistic compact objects and configurations with anisotropic matter distribution. The polytropic equation of state shall produce models associated to observed stellar objects and it makes our future works to further study in the upcoming papers.

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