

# NUMERICAL INVESTIGATIONS OF MHD FLOW PAST A SEMI-INFINITE VERTICAL PLATE WITH INCLINED MAGNETIC FIELD

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## ABSTRACT

The numerical investigations of unsteady MHD flow past a rotating semi-infinite plate with inclined magnetic field are presented in this paper. The fluid under consideration is considered to be electrically conducting. A variable magnetic field is subjected to the fluid at an inclined angle of  $\alpha$  in the z-axis in the  $xy$  plane. The governing equations that model the MHD problem are nonlinear hence the explicit finite difference method is used to discretise them coupled with the Gauss Seidel iteration technique. The equations are then non-dimensionalised together with the boundary conditions using various dimensionless parameters and numbers. Various flow parameters such as Prandtl number, Grashof number, magnetic parameter, Hall parameter, magnetic Reynolds number, Hartman number, Joule heating parameter, Eckert number and Schmidt number are varied and their effects on velocity, temperature and species concentration are investigated and discussed. It is noted that these parameters have significant effect on the velocity, temperature and species concentration profiles and mass transfer of the fluid.

**Keywords:** *Shall current, Skin friction, Magnetic.*

## 1. INTRODUCTION

The magnetohydrodynamic (MHD) flow of an electrically conducting fluid over a rotating plate has attracted attention due its application in the manufacturing processes such as polymer extrusion, wire drawing, hot rolling, paper production, metal spinning and glass fibre [1]. This is because a fluid that is electrically conducting that is subject to magnetic field is important because it can control the rate of cooling of a system. MHD problems have also attracted a lot of attention in the field of engineering such as geothermal energy extractions, plasma studies, MHD generators and nuclear reactors due to the effect of the magnetic field. This is due to the force that is generated in MHD flows under an electrically conducting fluid [2]. A lot of studies have studied MHD for horizontal and vertical plates. Liu [3] was able to analyse the effects of the magnetic field with heat and mass transfer over a stretching surface. Makinde [4] investigated MHD flow over a vertical heated plate under convective boundary conditions. Many industrial and engineering applications and processes occur under high temperatures and hence the knowledge of thermal effects is important in the design and usage [5]. The high temperatures during operations, the electrically conducting fluid is ionised during the radiative heat transfer [6]. Hence, the effect of thermal effects and heat transfer on MHD is an important aspect to be considered in research. Seth *et al.* [7] was able to investigate MHD flow past a rotating system in presence of magnetic field that was inclined. The results of their study noted that the angle of inclination accelerated the velocity of the fluid. Kinyanjui *et al.* [8] investigated the MHD flow past a rotating vertical plate with Hall current. They noted that velocity, heat transfer and concentration of the fluid was affected by the magnetic field. Nandkeolyar *et al.* [9] on the solutions of unsteady MHD fluid past a flat surface noted that the concentration increased with time but reduced with increase in Schmidt number and the temperature increased with increase in the thermal diffusion. Seth *et al.* [10] in their research were able to obtain an exact solution for an included magnetic field under the effects of hall current on a rotating flow. The combined effects of mass diffusion and thermal effects for unsteady flow of a viscous incompressible fluid over a semi-infinite porous plate was also investigate by Ahmed *et al.* [11] and Eldabe *et al.* [12]. Sharma *et al.* [13] studied one dimensional flow past a moving vertical plate in the presence of inclined magnetic field and noted that the velocity profile was affected by different flow parameters such as Hartman, Prandtl and Grashof number. Prasad *et al.* [14] researched on MHD flow and thermal effects of magnetic field and Hall current of an electrically conducting fluid with variable thickness over a stretching sheet. The governing equations in their study were solved using the explicit finite difference method. They noted that the magnetic field and Hall current had strong effects on heat transfer and the flow of the fluid. Maswai *et al.* [15] solved the governing equations using the finite difference method on their research on MHD turbulent flow past a rotating semi-infinite plate with an inclined magnetic field. Iva *et al.* [16] investigated MHD free convection over a vertical plate that was not inclined noting the effects of hall current, heat source and suction. They noted that the velocity, temperature and concentration distributions were dependent on the flow parameters that were varied. Ngesa *et al.* [17] investigated MHD heat and mass transfer past a semi-infinite porous plate and noted that various flow parameters influenced the velocity, concentration and temperature profiles.

Moreover, they solved the non-linear partial differential equations using the explicit difference approximation method. Bulinda *et al.* [18] studied MHD free convection of incompressible fluids with hall current, heat and mass transfer. Their research noted that velocity, concentration and temperature had significant effects on the fluid flow, the heat and mass transfer.

In spite of studies in MHD flows, the unsteady flow past a semi-infinite rotating vertical plate in presences of inclined magnetic field at an angle with effects of viscous dissipation, species concentration and joule heating has received little attention. Hence, the objective of the present study is to consider the effects of an inclined magnetic field taking into account the effects of viscous dissipation, species concentration and joule heating. The non-linear MHD equations are discretised using the explicit finite difference method coupled with the Gauss iteration method. The effects of different parameters on flow velocity, species concentration and temperature of the fluid are simulated and discussed in detail. To the best of my knowledge, the proposed MHD problem and results are new and have not been published anywhere.

## 2. MATHEMATICAL ANALYSIS

The MHD problem is a two-dimensional unsteady incompressible flow that considers the fluid is electrically conducting past a rotating plate under the influence of inclined magnetic field and heat transfer. The effects of Hall current and the induced magnetic field are assumed to be negligible and the Reynolds number is assumed to be small due to the electric intensity considered as zero [19]. The plate is heated by convection by a hot fluid of temperature  $T_w$ . The geometry of the MHD problem is given by Figure 1. The fluid is subjected to an inclined magnetic field that is variable at an angle of  $\alpha$  with the positive x axis in the xz plane. The system is made to rotate with uniform angular velocity  $\Omega$  in the presence of magnetic field that is varied. The magnetic field that is applied to the model is strong enough to generate the Hall current [8]. The effects of induction are also generated to the sufficient magnitude of the magnetic Reynolds number.

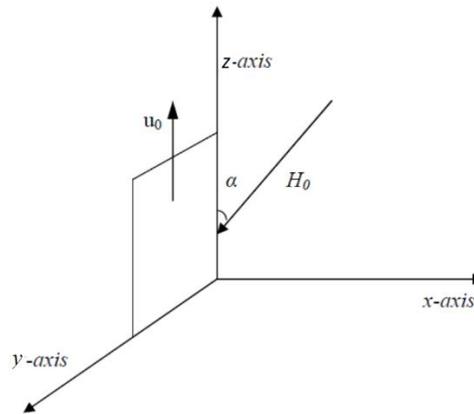


Figure 1. Flow configuration and coordinate system.

The continuity, momentum, energy and species concentration equations are given under Boussinesq boundary layer approximations. They are given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} - u_0 \frac{\partial u}{\partial z} + u \frac{\partial u}{\partial x} + 2\Omega v = g\beta(T - T_\infty) + \vartheta \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + \beta g(C - C_\infty) + \frac{\sigma \mu_0^2 H_0^2 \sin^2 \alpha (m(\sin \alpha) v - u)}{\rho(1+m^2 \sin^2 \alpha)}. \quad (2)$$

$$\frac{\partial v}{\partial t} - u_0 \frac{\partial v}{\partial z} + u \frac{\partial v}{\partial x} + 2\Omega u = \vartheta \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right) + \frac{\sigma \mu_0^2 H_0^2 \sin^2 \alpha (m(\sin \alpha) u + v)}{\rho(1+m^2 \sin^2 \alpha)}. \quad (3)$$

$$\rho C_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + u_0 \frac{\partial T}{\partial z} \right] = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right] + \vartheta \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] + \sigma \mu_0^2 H_0^2 \sin^2 \alpha (u^2 + v^2). \quad (4)$$

$$\frac{\partial C}{\partial t} + u_0 \frac{\partial C}{\partial x} + u \frac{\partial C}{\partial z} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2} \right). \quad (5)$$

where  $u, v$  are velocity components,  $x, z$  cartesian coordinates, time  $t$ , plate velocity  $u_0$ , gravitational acceleration  $g$ , volumetric expansion  $\beta$ , temperature  $T$ , free stream temperature  $T_\infty$ , kinematic viscosity  $\vartheta$ , density of the fluid  $\rho$ , species concentration  $C$ , species concentration at plate  $C_w$ , species concentration at free stream  $C_\infty$ , electrical conductivity  $\sigma$ , magnetic permeability  $\mu_0$ , magnetic field strength  $H_0$ , Hall parameter  $m$ , specific heat constant pressure  $C_p$ , thermal conductivity  $k$  and diffusion coefficient  $D$ .

The initial and boundary conditions for the unsteady MHD problem are given by,

$$\begin{aligned} t \leq 0, \quad u(x, z, t) = 0, \quad v(x, z, t) = 0, \quad T(x, z, t) = 0, \quad C(x, z, t) = 0 \\ t > 0, \quad u(0, z, t) = u_0, \quad v(0, z, t) = 0, \quad T(0, z, t) = 0, \quad C(x, z, t) = C_\infty \\ t > 0, \quad u(\infty, z, t) = 0, \quad v(\infty, z, t) = 0, \quad T(\infty, z, t) = 0, \quad C(\infty, z, t) = 0 \end{aligned} \quad (6)$$

The following dimensionless variables and quantities are introduced into the continuity, momentum, energy balance and species concentration equations. That is Eckert number  $E_c$ , Dimensionless temperature  $\theta$ , Prandtl number  $P_r$ , Joule heating parameter  $R$ , Hartman number  $M$ , Grashof number  $G_r$ , Modified Grashof number  $G_c$ , Magnetic Reynolds number  $R_m$  Rotation parameter  $E_r$ , Schmidt number  $S_c$  and dimensionless species concentration  $\bar{C}$ .

$$\begin{aligned} \bar{x} = \frac{x u_0}{u}, \quad \bar{z} = \frac{z u_0}{u}, \quad \bar{t} = \frac{t u_0^2}{u}, \quad \bar{u} = \frac{u}{u_0}, \quad \bar{v} = \frac{v}{u_0} \\ E_c = \frac{u_0^3}{(T_w - T_\infty) C_p}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad P_r = \frac{\mu_e C_p}{k}, \quad R = \frac{H_0 \sigma}{\rho C_p} \\ G_r = \frac{g \beta \theta (T_w - T_\infty)}{u_0^3}, \quad M^2 = \frac{\sigma \mu_e H_0^2 \vartheta}{u_0^2 \rho}, \quad G_c = \frac{g \beta \vartheta (C - C_\infty)}{u_0^3} \\ R_m = \mu_e u_0 \sigma, \quad E_r = \frac{\Omega \vartheta}{u_0^2}, \quad S_c = \frac{D}{\vartheta}, \quad \bar{C} = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \quad (7)$$

The resulting equations become;

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{z}} = 0. \quad (8)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} - u_0 \frac{\partial \bar{u}}{\partial \bar{z}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} - 2E_r \bar{v} = \theta G_r + \left( \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) + G_c \bar{C} + M^2 \sin^2 \alpha \left( \frac{m(\sin \alpha) \bar{v} - \bar{u}}{(1 + m^2 \sin^2 \alpha)} \right). \quad (9)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} - u_0 \frac{\partial \bar{v}}{\partial \bar{z}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + 2E_r \bar{u} = \left( \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right) - M^2 \sin^2 \alpha \left( \frac{m(\sin \alpha) \bar{v} + \bar{u}}{(1 + m^2 \sin^2 \alpha)} \right). \quad (10)$$

$$\frac{\partial \theta}{\partial \bar{t}} + \bar{u} \frac{\partial \theta}{\partial \bar{x}} + u_0 \frac{\partial \theta}{\partial \bar{z}} = \frac{1}{P_r} \left[ \frac{\partial^2 \theta}{\partial \bar{x}^2} + \frac{\partial^2 \theta}{\partial \bar{z}^2} \right] + E_c \left[ \left( \frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + \left( \frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 \right] + R \sin^2 \alpha (\bar{u}^2 + \bar{v}^2). \quad (11)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + u_0 \frac{\partial \bar{C}}{\partial \bar{z}} = S_c \left( \frac{\partial^2 \bar{C}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} \right). \quad (12)$$

with dimensionless initial and boundary conditions,

$$\begin{aligned} \bar{t} \leq 0, \quad \bar{u}(\bar{x}, \bar{z}, \bar{t}) = 0, \quad \bar{v}(\bar{x}, \bar{z}, \bar{t}) = 0, \quad \theta(\bar{x}, \bar{z}, \bar{t}) = 0, \quad \bar{C}(\bar{x}, \bar{z}, \bar{t}) = 0 \\ \bar{t} > 0, \quad \bar{u}(0, \bar{z}, \bar{t}) = 1, \quad \bar{v}(0, \bar{z}, \bar{t}) = 1, \quad \theta(0, \bar{z}, \bar{t}) = 0, \quad \bar{C}(\bar{x}, \bar{z}, \bar{t}) = 1 \\ \bar{t} > 0, \quad \bar{u}(\infty, \bar{z}, \bar{t}) = 0, \quad \bar{v}(\infty, \bar{z}, \bar{t}) = 0, \quad \theta(\infty, \bar{z}, \bar{t}) = 0, \quad \bar{C}(\infty, \bar{z}, \bar{t}) = 0 \end{aligned} \quad (13)$$

### 3. NUMERICAL PROCEDURE

The explicit finite difference method is used to discretise the governing equations since they are non-linear together with their initial and boundary conditions. The discretization is usually based on a uniform grid and linear mesh on the cartesian plane. A spatial interval  $0 \leq dx \leq n_{max}$  partition is introduced in the x and z axis. This partition is then subdivided into N equal parts with grid sizes  $dx = dz = dt = 1/N$  and  $dz = 1/N$  and grid points  $\bar{x}_i = (i - 1)dx, 1 \leq i \leq N + 1, \bar{z}_j = (j - 1)dz, 1 \leq j \leq N + 1$  and  $\bar{t}_k = (k - 1)dt, 1 \leq k \leq N + 1$ .

The first and second order partial derivatives are usually approximated with central finite differences. The discrete system of equations for the MHD problem becomes

$$\bar{u}_{i,j}^k = \left( \frac{M^2 \sin^2 \alpha}{1+m^2 \sin^2 \alpha} + \frac{2}{(dx)^2} + \frac{2}{(dz)^2} \right)^{-1} \left[ \frac{-1}{2dx} \left( (\bar{u}_{i+1,j}^k)^2 - (\bar{u}_{i-1,j}^k)^2 \right) + \frac{1}{(dx)^2} (\bar{u}_{i+1,j}^k - \bar{u}_{i-1,j}^k) - \frac{u_0}{2dz} (\bar{u}_{i,j+1}^k - \bar{u}_{i,j-1}^k) + \frac{1}{(dz)^2} (\bar{u}_{i,j+1}^k - \bar{u}_{i,j-1}^k) - 2E_r \bar{u}_{i,j}^k - M^2 \sin^2 \alpha \left( \frac{m(\sin \alpha) \bar{u}_{i,j}^k}{1+m^2 \sin^2 \alpha} \right) \right] \tag{14}$$

$$\bar{v}_{i,j}^k = \left( \frac{M^2 \sin^2 \alpha}{1+m^2 \sin^2 \alpha} + \frac{2}{(dx)^2} + \frac{2}{(dz)^2} \right)^{-1} \left[ \frac{-1}{2dx} (\bar{v}_{i+1,j}^k \bar{u}_{i+1,j}^k - \bar{v}_{i-1,j}^k \bar{u}_{i-1,j}^k) + \frac{1}{(dx)^2} (\bar{v}_{i+1,j}^k - \bar{v}_{i-1,j}^k) - \frac{u_0}{2dz} (\bar{v}_{i,j+1}^k - \bar{v}_{i,j-1}^k) + \frac{1}{(dz)^2} (\bar{v}_{i,j+1}^k - \bar{v}_{i,j-1}^k) + \theta G_r + 2E_r \bar{v}_{i,j}^k + M^2 \sin^2 \alpha \left( \frac{m(\sin \alpha) \bar{v}_{i,j}^k}{1+m^2 \sin^2 \alpha} \right) \right] \tag{15}$$

$$\theta_{i,j}^k = \left[ \frac{Pr(dx)^2(dz)^2}{2(dx)^2+2(dz)^2} \right] \left[ \frac{-1}{2dx} (\theta_{i+1,j}^k \bar{u}_{i+1,j}^k - \theta_{i-1,j}^k \bar{u}_{i-1,j}^k) - \frac{u_0}{2dz} (\theta_{i,j+1}^k - \theta_{i,j-1}^k) + \frac{Ec}{4} \left( \frac{(\bar{u}_{i+1,j}^k + \bar{u}_{i-1,j}^k)^2}{(dx)^2} + \frac{(\bar{v}_{i,j+1}^k + \bar{v}_{i,j-1}^k)^2}{(dz)^2} \right) + \frac{1}{Pr} \left( \frac{\theta_{i+1,j}^k + \theta_{i-1,j}^k}{(dx)^2} + \frac{\theta_{i,j+1}^k + \theta_{i,j-1}^k}{(dz)^2} \right) + R \sin^2 \alpha ((\bar{u}_{i,j}^k)^2 + (\bar{v}_{i,j}^k)^2) \right] \tag{16}$$

$$\bar{C}_{i,j}^k = \left[ \frac{Sc}{(dx)^2} + \frac{Sc}{(dz)^2} - \frac{1}{dt} \right]^{-1} \left[ - \left( \frac{\bar{C}_{i,j+1}^k - \bar{C}_{i,j-1}^k}{2dx} \right) - \left( \frac{\bar{C}_{i,j+1}^k - \bar{C}_{i,j-1}^k}{2dz} \right) + Sc \left( \frac{\bar{C}_{i+1,j}^k + \bar{C}_{i+1,j}^k}{(dx)^2} + \frac{\bar{C}_{i,j-1}^k + \bar{C}_{i,j-1}^k}{(dx)^2} \right) - \frac{\bar{C}_{i,j}^{k+1}}{dt} \right] \tag{17}$$

### 4. RESULTS AND DISCUSSION

The numerical solutions for the unsteady MHD problem are obtained for various parameters. The analysis and investigation of the MHD model involves generating velocity, temperature and species concentration profiles for varied values of Prandtl number, Grashof number, inclination angle, Hall parameter, Eckert number, Hartman number, Joule heating parameter, magnetic Reynolds number and Schmidt number.

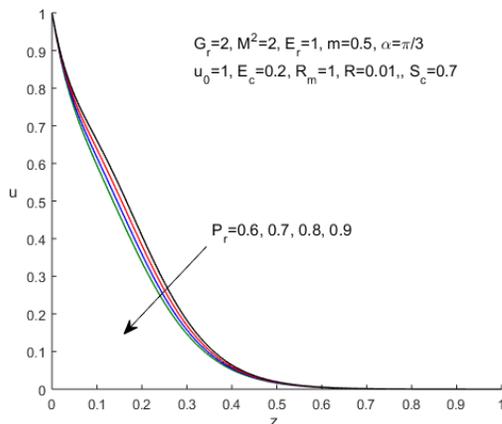


Figure 2. Velocity profiles for various values of  $P_r$ .

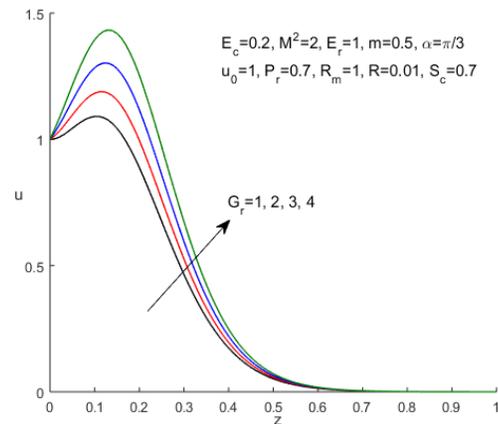
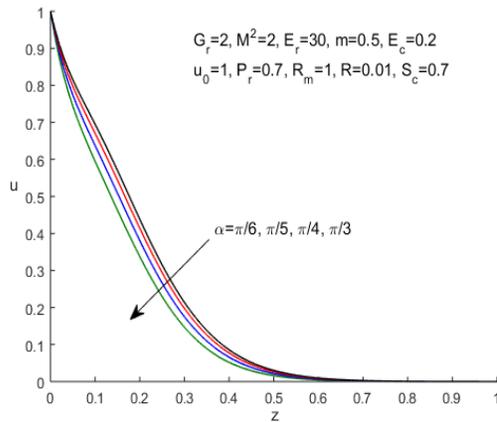
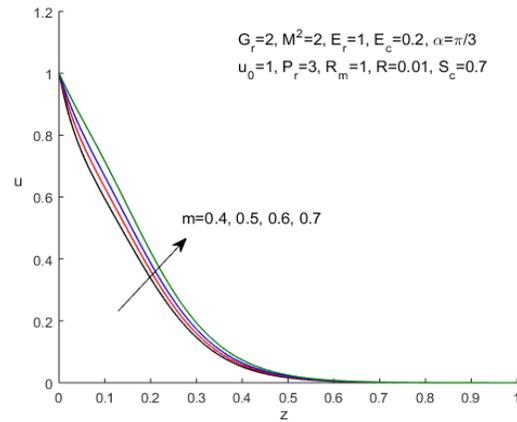
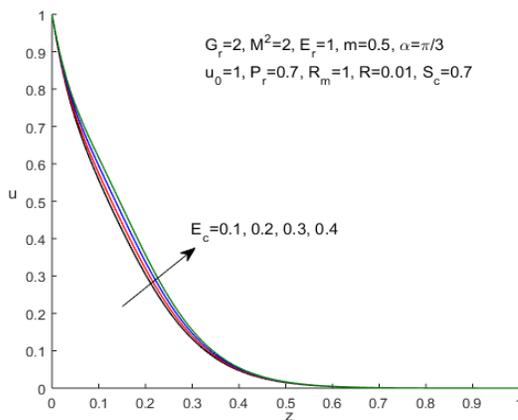
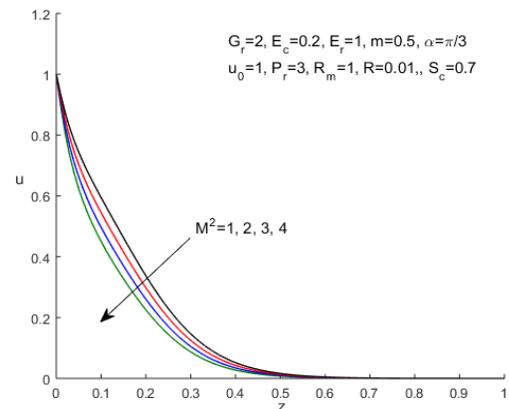


Figure 3. Velocity profiles for various values of  $G_r$ .

Figure 4. Velocity profiles for various values of angle  $\alpha$ .Figure 5. Velocity profiles for various values of  $m$ .Figure 6. Velocity profiles for various values of  $E_c$ .Figure 7. Velocity profiles for various values of  $M^2$ .

#### 4.1. Effects of parameter variation on velocity profiles

Figures 2-7 illustrate the effects of various parameters on the transient velocity profiles. It is noted from Figure 2 that increasing the Prandtl number leads to a decrease in the velocity of the fluid. The Prandtl number is usually a ratio of the kinematic viscosity to thermal diffusivity of the fluid. Thus, increasing the Prandtl number results to an increase in the kinematic viscosity of the fluid which increases its viscosity. The increase in viscosity results to resistance in flow of the fluid which reduces its velocity. Figure 3 illustrates the effects of Grashof number. The Grashof number usually the ratio of the thermal buoyancy force to viscous hydrodynamic force in the flow of the boundary layer. The transient velocity profiles increase with increase in Grashof number. The velocity of the fluid increases reaching a peak value near the surface of the plate and then decreases to zero satisfying the far field condition. This is a result of increase in the thermal buoyance force. The transient velocity profiles with effect of angle of inclination of the magnetic field are represented by. It is observed that an increase in the angle of inclination of the magnetic field results to a decrease in the transient velocity. The increase in angle reduces the magnetic field intensity due to increase in the Lorentz force. The Lorentz force leads to resistance in the fluid flow thereby reducing the velocity of the fluid. Figure 5 shows the effects of Hall parameter on the transient velocity profiles of the fluid. It is observed as the Hall parameter increases; the transient velocity profiles also increase. An increase in the Hall parameter leads to a decrease in the effective conductivity which reduces the magnetic damping force. This reduction in the magnetic damping force reduces the resistance to flow of the fluid which in turn increases its velocity. The effects of Eckert number on the transient velocity profiles are illustrated by Figure 6. An increase in Eckert number leads to an increase in viscous heating of the fluid. This makes the fluid lighter since of reduction in the viscosity, hence the fluid flows faster. The effects of the Hartman number on the transient velocity profiles are illustrated in Figure 7. The velocity decreases with increase in Hartman number. An increase in Hartman number increases the Lorentz force that generated is usually by the magnetic field. This force normally acts against the flow

of the fluid which leads to a decrease in its momentum. The transient velocity profiles observation in this study when different parameters are varied are in good agreement with earlier results as reported by Adem [21].

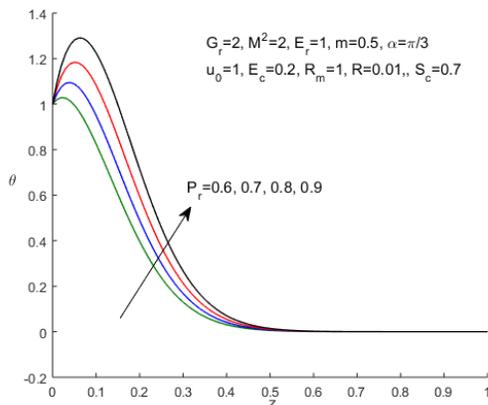


Figure 8. Temperature profiles for various values of  $P_r$ .

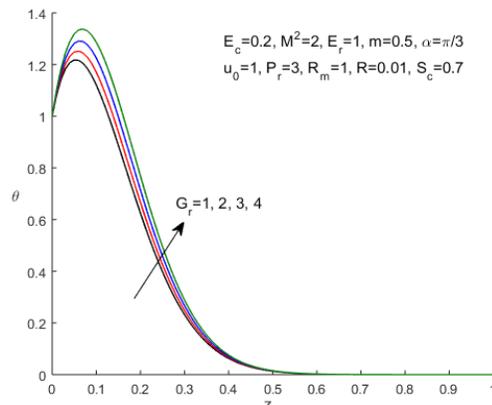


Figure 9. Temperature profiles for various values of  $G_r$ .

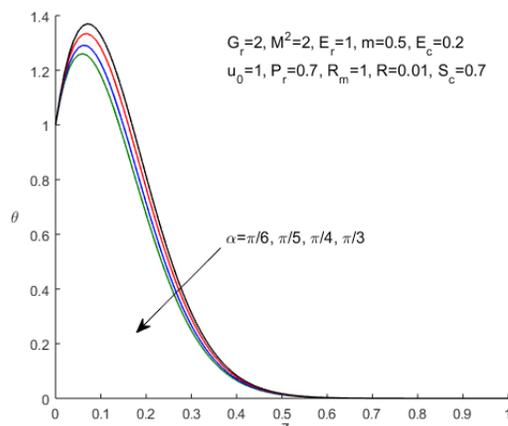


Figure 10. Temperature profiles for various values of  $\alpha$ .

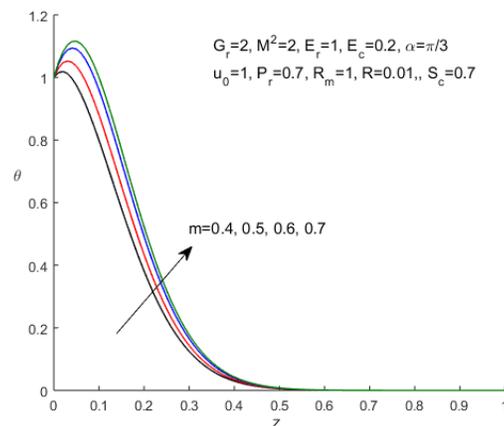


Figure 11. Temperature profiles for various values of  $m$ .

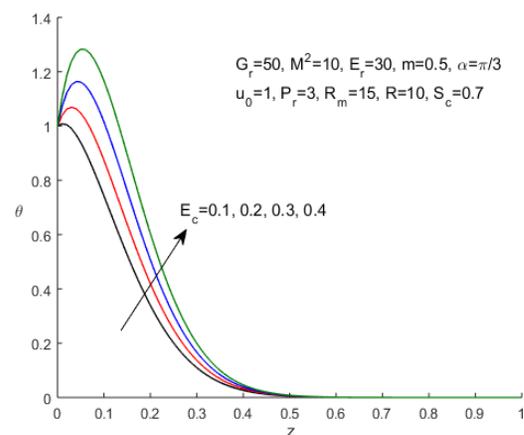


Figure 12. Temperature profiles for various values of  $E_c$ .

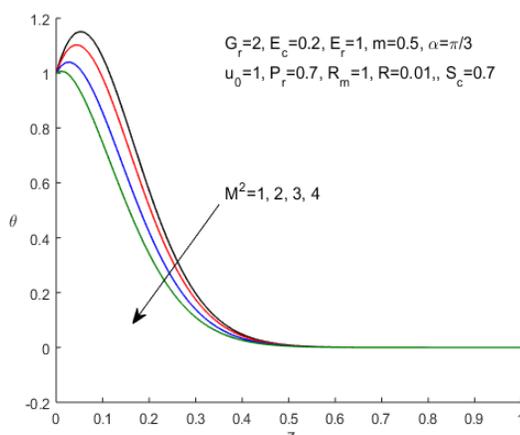


Figure 13. Temperature profiles for various values of  $M^2$ .

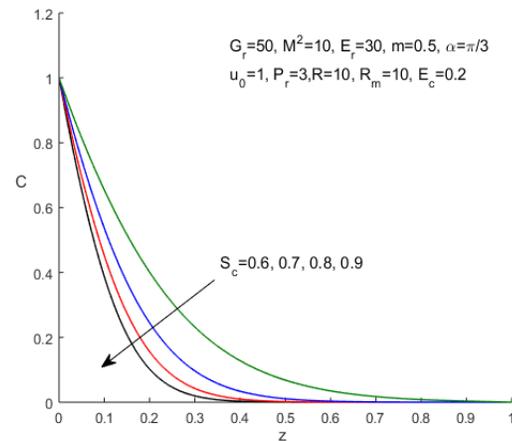
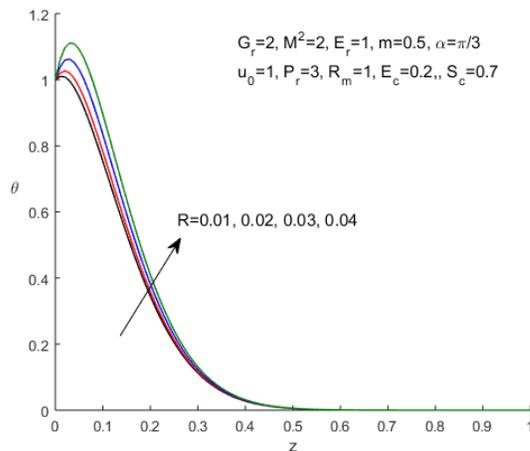


Figure 14. Temperature profiles for various values of  $R$ . Figure 15. Concentration profiles for various values of  $S_c$ .

#### 4.2. Effects of parameter variation on temperature profiles

Figures 8-14 demonstrate the temperature profiles for different variations of the parameters. It is observed from all these figures that the temperature increases to a certain point, reaching its peak near the surface of the plate and then decreases to zero at free stream which satisfying the boundary conditions. Figure 8 shows the effects of the Prandtl number on the temperature profiles. It is noted that temperature of the fluid reduces when the Prandtl number is increased. An increase in the Prandtl number results to a decrease in the thermal conductivity. This makes the heat to diffuse away from the surface of the plate that is heated. This results to less heat reaching the fluid which attributes to temperature decline. Figure 9 illustrates the effects of Grashof number on the temperature profiles. It is noted that an increase in the Grashof number results to increase in the temperature of the fluid. Increasing the Grashof number increases the thermal buoyancy force which makes heat to be conducted better from the plate into the fluid. Figure 10 illustrates the effects of the inclination angle of the magnetic field on the fluid temperature. Increasing the inclination angle of the magnetic field results to a reduction in the fluid temperature. Increasing the angle of inclination of the magnetic field reduces the rate of heat transfer thereby thickening the thermal boundary layer which in turn reduces the rate of heat transfer. This results to a reduction in fluid temperature. Figure 11 illustrates the effects of Hall parameter on the temperature of the fluid. It is noted that the temperature of the fluid increases with increase in the Hall parameter. Increasing the Hall parameter decreases the thermal boundary layer thickness. This means the temperature gradient reduces and makes heat flow easily which in turn increases the fluid temperature. Figure 12 illustrates the effects of Eckert number on the temperature profiles. It is noted that the temperature of the fluid increases with increase in Eckert number. The increase in the Eckert number results to viscous heating which constitutes to additional internal heat generation. This additional heat generation results to elevation of the temperature of the fluid. Figure 3 illustrates the effect of Hartman number on the temperature of the fluid. The temperature of the fluid reduces with increase in Hartman number. The increase in Hartman number increases the Lorentz force. This force reduces the thermal viscous dissipation in the fluid resulting to a reduction in the fluid's temperature. Figure 14 illustrates the effects of Joule heating parameter on the temperature profiles. Increasing the Joule heating parameter results to an increase in the fluid's temperature. Increasing the Joule heating Parameter means more current is applied on the plate which leads to more heating of the fluid thereby increasing its temperature. The temperature profiles are in good agreement with earlier results as reported by Adem [20] when different parameters are varied

#### 4.3. Effects of parameter variation on concentration profile

Figure 15 shows the effects of Schmidt number on the species concentration of the fluid. It is noted that the species concentration of the fluid reduces with increase in Schmidt number. The Schmidt number is the ratio of the viscosity of the fluid to the mass diffusivity. An increase in the Schmidt number results to a decrease in the mass diffusivity rate of the fluid. This means a reduction in the concentration of boundary layer which leads to overall reduction in the species concentration. The species concentration profiles in this study with effects of Schmidt number are in good agreement with earlier results as reported by Adem [20].

### 5. CONCLUSIONS

A numerical analysis is investigated to the study of the unsteady MHD flow of fluid past a rotating semi-infinite vertical plate with inclined magnetic field. The non-linear equations that arise in the model are solved numerically using the finite difference method coupled with the Gauss Seidel iteration technique. Some of the notable conclusions of the study are as follows.

- (i) The transient velocity profiles increase with increase in Prandtl number, Grashof number, Hall parameter and Eckert number. However, they decrease with increase in the angle of inclination of the magnetic field and Hartman number.
- (ii) The temperature profiles increase with increase in Grashof number, Hall parameter, Eckert number and Joule heating parameter. However, they decrease with increase in Prandtl number, angle of inclination of magnetic field and Hartman number.
- (iii) The species concentration profiles decrease with increase in the Schmidt number.

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