EVALUATION OF RELIABILITY FACTORS USING BOOLEAN FUNCTION TECHNIQUE IN MILK POWDER MANUFACTURING PLANT

Shikha Bansal[1, *], S.C. Agarwal[2] & Kuldeep Sharma[3]
[1, 3]Lecturer, Dept. of mathematics, Krishna Institute of Engineering and Technology, Ghaziabad, India
[2]Reader, Department of Mathematics, M.M (P.G) College, Modinagar, India
*E-mail: srbansal2008@gmail.com

ABSTRACT

For systems of multistate elements, the problem of developing Boolean reliability models was considered on the basis of logic, algebra of groups of incompatible events and classical logic and probabilistic method. The objective of this paper is to compute the terminal reliability of milk powder manufacturer plant based on a minimizing Boolean expression technique. Milk powder consists of four subsystems A, B, C, D viz; storage, hot plates, evaporator, dryer, arranged in series respectively. Subsystem A and C has two units in standby with perfect switching. Subsystem B has two units in parallel redundancy and subsystem D has one unit. The failure rate is exponentially distributed. The reliability and MTTF have been evaluated. We demonstrate the use our technique by means of examples and present numerical results to show the effects of mission phases on the system reliability.

Keywords: *Boolean function, Reliability, MTTF, Redundancy, Perfect switching*

1. INTRODUCTION

The development of science and technology and the needs of modern society are racing against each other. Industries are trying to introduce more automation in their industrial processes in order to meet the ever increasing demands of the society. The complexity of industrial systems, as well as their products, is increasing day-by-day. The improvement in effectiveness of such complex system has therefore, acquired special attention in recent years. The effectiveness of a system is understood to mean the suitability of the system for the fulfillment of the intended task and the efficiency of utilizing the means put into it. The suitability of performing definite tasks is primarily determined by the reliability of the systems. The need of obtaining highly reliable systems has acquired special importance with the development of the present-day technology.

Reliability engineering has wide applications in milk powder process. Reliability is an important concept at the planning design and operation stages of various transit systems.

[4, 5, 7, 8] have calculated the reliability of several electronic equipments using various techniques but the method adopted by them lead to cumbersome and tedious calculation. Keeping this fact in view the authors have applied Boolean function technique for the evaluation of various factors of reliability in milk powder plant. Milk powder manufacture is a simple process now carried out on a large scale. In Milk manufacturing process, whole milk to make a variety of powders, including whole milk, skimmed milk, butter milk, as well as their protein powder. A High quality powdered product will provide the original qualities of milk when it is reconstituted with water. The basic steps are shown in fig 1.

Fig-1

The milk powder plant under consideration consist of four subsystems; viz; A, B, C, D. Subsystem A consist of two milk storage in standby arrangements i.e. when the main unit ‘x_1’.in action is non-operable, the identical unit ‘x_3’ is put into operation through switch ‘x_2’. Milk powder plant manufacture is a simple process includes the gentle removal of water at the lowest possible cost. During this process the water is removed by boiling the milk under reduced pressure at low temperature (known as evaporation). This whole process is carried out through subsystems B and C. subsystem B consists of two hot plates x_4 and x_5 in parallel and C has evaporators x_6 and x_8 in standby mode with perfect switching x_7.

As at this there is still another step, the milk powder is still almost half of water. Drying helps eliminate the extra water, and large scale milk processing plants are dryers**.** Drying involves atomising the milk concentrate from the evaporator into fine droplets; this step lowers the moisture content from just under half. The block diagram of the system is shown in Fig2

Several techniques for determining reliability factors of the system are available in literature. Reliability means probability of success between two units connected by several branches knowing the reliability of each branch, one can determine the terminal reliability between any two units, the Boolean algebra approach has been used by several investigators [1,2] to determine the terminal reliability of a system. The Boolean algebra technique basically consists of following steps:

(1) Determine simple paths between units of a graph.
(2) Write down the Boolean expression corresponding to the paths where the Boolean variables correspond to the different units
(3) Determine a disjoint expression corresponding to the Boolean expression given in step 2.
(4) Given the disjoint Boolean expression substitute the corresponding values of probabilities to get the terminal reliability.

These 4 steps algorithm is to minimize or to obtain the Boolean expression and then compute the reliability of the system.

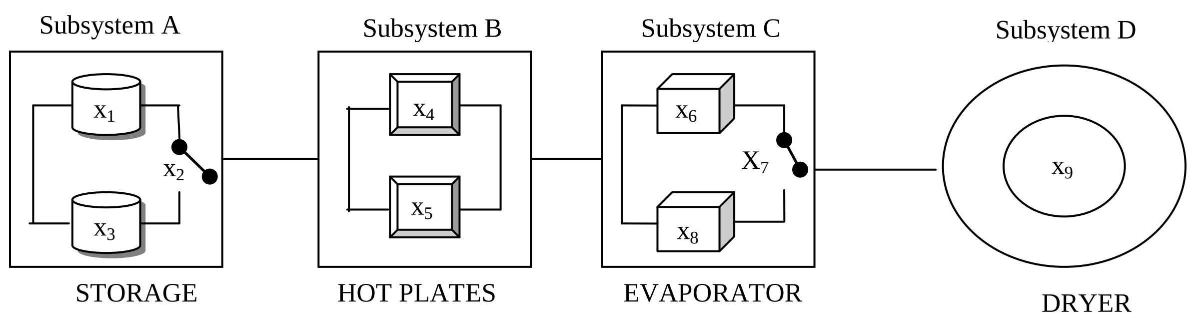

Fig-2: Block Diagram

2. ASSUMPTIONS

The following assumptions have been associated with this model:

a) Initially, the whole system is good and operable.
b) Every component of the system remains either in good or in bad state.
c) There is no repair facility to repair a failed component.
d) The reliability of the each part moreover each section is in advance.
e) The whole system can fail due to failure of any one of its units
f) Failures are statistically-independent.

3. NOTATIONS

x_1, x_3	:	States of tanks
x_2, x_7	:	States of perfect Switching devices
x_4, x_5	:	States of heater.
x_6, x_8	:	States of vapor separator
x_9	:	States of dryer
$x_i(i = 1,2 - -9)$	:	1 in good state; 0 is bad state.
x_i'	:	Negation of for x_i' all i
$\wedge / \vee$	:	Conjunction / Disjunction.

$\lvert\ \rvert$	:	This notation has used to represent logical matrix.
R_i	:	Reliability of i^{th} part of the system, $\forall i = 1,2 - -9$.
Q_i	:	$1-R_i$
R_S	:	Reliability of the whole system.
$R_{SW}(t)/R_{SE}(t)$	:	Reliability of the system as a whole when failures follow Weibull / Exponential time distribution.

4. FORMULATION OF MATHEMATICAL MODEL

By making use of Boolean function technique, the conditions of capability of successful operation of the system in terms of logical matrix are expressed as shown below:

$$F(x_1, x_2, ----, x_9) = \begin{vmatrix} x_1 & x_4 & x_6 & x_9 & & \\ x_1 & x_4 & x_7 & x_8 & x_9 & \\ x_1 & x_5 & x_6 & x_9 & & \\ x_1 & x_5 & x_7 & x_8 & x_9 & \\ x_2 & x_3 & x_4 & x_6 & x_9 & \\ x_2 & x_3 & x_4 & x_7 & x_8 & x_9 \\ x_2 & x_3 & x_5 & x_6 & x_9 & \\ x_2 & x_3 & x_5 & x_7 & x_8 & x_9 \end{vmatrix} \quad \ldots (1)$$

5. SOLUTION OF THE MODEL

By using algebra of logics, we may write equation (1) again as,

$$F(x_1, x_2, ----, x_9) = \left| x_9 \wedge f(x_1, x_2, ----, x_9) \right| \quad \ldots (2)$$

Where,

$$f(x_1, x_2, ----, x_9) = \begin{vmatrix} x_1 & x_4 & x_6 & & \\ x_1 & x_4 & x_7 & x_8 & \\ x_1 & x_5 & x_6 & & \\ x_1 & x_5 & x_7 & x_8 & \\ x_2 & x_3 & x_4 & x_6 & \\ x_2 & x_3 & x_4 & x_7 & x_8 \\ x_2 & x_3 & x_5 & x_6 & \\ x_2 & x_3 & x_5 & x_7 & x_8 \end{vmatrix} = \begin{vmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \end{vmatrix} \quad \ldots (3)$$

where,

$$M_1 = \begin{vmatrix} x_1 & x_4 & x_6 \end{vmatrix} \quad \ldots (4)$$

$$M_2 = \begin{vmatrix} x_1 & x_4 & x_7 & x_8 \end{vmatrix} \quad \ldots (5)$$

$$M_3 = \begin{vmatrix} x_1 & x_5 & x_6 \end{vmatrix} \quad \ldots (6)$$

$$M_4 = \begin{vmatrix} x_1 & x_5 & x_7 & x_8 \end{vmatrix} \quad \ldots (7)$$

$$M_5 = \begin{vmatrix} x_2 & x_3 & x_4 & x_6 \end{vmatrix} \quad \ldots (8)$$

$$M_6 = \left| x_2 \quad x_3 \quad x_4 \quad x_7 \quad x_8 \right| \qquad \dots (9)$$

$$M_7 = \left| x_2 \quad x_3 \quad x_5 \quad x_6 \right| \qquad \dots (10)$$

$$M_8 = \left| x_2 \quad x_3 \quad x_5 \quad x_7 \quad x_8 \right| \qquad \dots (11)$$

By orthogonalization algorithm (3) may be written as

$$f(x_1, x_2, ----, x_9) = \begin{vmatrix} M_1 \\ M_1' \quad M_2 \\ M_1' \quad M_2' \quad M_3 \\ M_1' \quad M_2' \quad M_3' \quad M_4 \\ M_1' \quad M_2' \quad M_3' \quad M_4' \quad M_5 \\ M_1' \quad M_2' \quad M_3' \quad M_4' \quad M_5' \quad M_6 \\ M_1' \quad M_2' \quad M_3' \quad M_4' \quad M_5' \quad M_6' \quad M_7 \\ M_1' \quad M_2' \quad M_3' \quad M_4' \quad M_5' \quad M_6' \quad M_7' \quad M_8 \end{vmatrix} \qquad \dots (12)$$

Now we $M_1{}' M_2 = \begin{vmatrix} x_1{}' \\ x_1 \quad x_4' \\ x_1 \quad x_4 \quad x_6{}' \end{vmatrix} \wedge \left| x_1 \quad x_4 \quad x_7 \quad x_8 \right|$

$$= \left| x_1 \quad x_4 \quad x_6{}' \quad x_7 \quad x_8 \right| \qquad \dots (13)$$

Similarly,

$$M_1{}' M_2{}' M_3 = \left| x_1 \quad x_4{}' \quad x_5 \quad x_6 \right| \qquad \dots (14)$$

$$M_1{}' M_2{}' M_3{}' M_4 = \left| x_1 \quad x_4' \quad x_5 \quad x_6' \quad x_7 \quad x_8 \right| \qquad \dots (15)$$

$$M_1{}' M_2{}' M_3{}' M_4{}' M_5 = \left| x_1{}' \quad x_2 \quad x_3 \quad x_4 \quad x_6 \right| \qquad \dots (16)$$

$$M_1{}^1 M_2{}^1 M_3{}^1 M_4{}^1 M_5{}^1 M_6 = \left| x_1{}^1 \quad x_2 \quad x_3 \quad x_4 \quad x_6{}^1 \quad x_7 \quad x_8 \right| \qquad \dots (17)$$

$$M_1' M_2' M_3' M_4' M_5' M_6' M_7 = \left| x_1' \quad x_2 \quad x_3 \quad x_4' \quad x_5 \quad x_6 \right| \qquad \dots (18)$$

$$M_1{}' M_2{}' M_3{}' M_4{}' M_5{}' M_6{}' M_7{}' M_8 = \left| x_1{}' \quad x_2 \quad x_3 \quad x_4{}' \quad x_5 \quad x_6{}' \quad x_7 \quad x_8 \right| \qquad ...(19)$$

Using all these values in equation (12),we obtain

$$f\left(x_1, x_2, ----, x_9\right) = \left| \begin{array}{llllllll} x_1 & x_4 & x_6 & & & & & \\ x_1 & x_4 & x_6' & x_7 & x_8 & & & \\ x_1 & x_4' & x_5 & x_6 & & & & \\ x_1 & x_4' & x_5 & x_6' & x_7 & x_8 & & \\ x_1' & x_2 & x_3 & x_4 & x_6 & & & \\ x_1^1 & x_2 & x_3 & x_4 & x_6^1 & x_7 & x_8 & \\ x_1' & x_2 & x_3 & x_4' & x_5 & x_6 & & \\ x_1' & x_2 & x_3 & x_4' & x_5 & x_6' & x_7 & x_8 \end{array} \right| \quad \ldots (20)$$

Using (20), equation (2) becomes

$$F\left(x_1, x_2, ----, x_9\right) = \left| \begin{array}{lllllllll} x_1 & x_4 & x_6 & x_9 & & & & & \\ x_1 & x_4 & x_6' & x_7 & x_8 & x_9 & & & \\ x_1 & x_4' & x_5 & x_6 & x_9 & & & & \\ x_1 & x_4' & x_5 & x_6' & x_7 & x_8 & x_9 & & \\ x_1' & x_2 & x_3 & x_4 & x_6 & x_9 & & & \\ x_1^1 & x_2 & x_3 & x_4 & x_6^1 & x_7 & x_8 & x_9 & \\ x_1' & x_2 & x_3 & x_4' & x_5 & x_6 & x_9 & & \\ x_1' & x_2 & x_3 & x_4' & x_5 & x_6' & x_7 & x_8 & x_9 \end{array} \right| \quad \ldots (21)$$

Since equation (20) is the disjunction of disjoint conjunctions, therefore, the reliability of the whole system is given by

$$R_S = P_r\left\{F(x_1, x_2 ....x_9) = 1\right\}$$

Or,

$$R_S = R_9\big[\ R_1R_4R_6 + R_1R_4Q_6R_7R_8 + R_1Q_4R_5R_6 + R_1Q_4R_5Q_6R_7R_8 + Q_1R_2R_3R_4R_6 \\ + Q_1R_2R_3R_4Q_6R_7R_8 + Q_1R_2R_3Q_4R_5R_6 + Q_1R_2R_3Q_4R_5R_6 + Q_1R_2R_3Q_4R_5Q_6R_7R_8\ \big]$$

Where R_i is the reliability corresponding to system state x_i

And Q_I=1-$R_{i,}$ $\forall$ i=1, 2…..9

Thus

$$R_S = R_9\big[R_1R_4R_6 + R_1R_4R_7R_8 - R_1R_4R_6R_7R_8 + R_1R_5R_6 - R_1R_4R_5R_6 + R_1R_5R_7R_8$$

$$-R_1R_4R_5R_7R_8 - R_1R_5R_6R_7R_8 + R_1R_4R_5R_6R_7R_8 + R_2R_3R_4R_6 - R_1R_2R_3R_4R_6$$

$$R_2R_3R_4R_7R_8 - R_1R_2R_3R_4R_7R_8 - R_2R_3R_4R_6R_7R_8 + R_1R_2R_3R_4R_6R_7R_8 + R_2R_3R_5R_6$$

$$- R_1R_2R_3R_5R_6 - R_2R_3R_4R_5R_6 + R_1R_2R_3R_4R_5R_6 + R_2R_3R_5R_7R_8 - R_1R_2R_3R_5R_7R_8$$

$$- R_2R_3R_4R_5R_7R_8 - R_2R_3R_5R_6R_7R_8 + R_1R_2R_3R_4R_5R_7R_8 + R_2R_3R_4R_5R_6R_7R_8$$

$$+ R_1R_2R_3R_5R_6R_7R_8 - R_1R_2R_3R_4R_5R_6R_7R_8] \quad \ldots (22)$$

Where, $R_i(i = 1,2 - -9)$ is the reliability of the section state $x_i(i = 1,2 - -9,)$respectively

6. SOME PARTICULAR CASES

Case I: when reliability of each component R

Then equation (22) yields:

$$R_S = 2R^4 + 3R^5 - 4R^6 - 3R^7 + 4R^8 - R^9 \quad \ldots (23)$$

Case II: when failure rates follow Weibull time distribution:

Let λ_i be the failure rate corresponding to system state $x_i, \forall i = 1,2 - -9,$, then reliability of considered system at an instant‘t’, is given by

$$R_{SW}(t) = \sum_{i=1}^{14} \exp.\{-\alpha_i t^p\} - \sum_{j=1}^{13} \exp.\{-\beta_j t^p\} \quad \ldots (24)$$

Where, p is a positive parameter and

$\alpha_1 = \lambda_1 + \lambda_4 + \lambda_6 + \lambda_9$

$\alpha_2 = \lambda_1 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_3 = \lambda_1 + \lambda_5 + \lambda_6 + \lambda_9$

$\alpha_4 = \lambda_1 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_5 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_6 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_9$

$\alpha_7 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_8 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_9 = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_9$

$\alpha_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$

$\alpha_{11} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_{12} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_{13} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\alpha_{14} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

and

$\beta_1 = \lambda_1 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_2 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$

$\beta_3 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_4 = \lambda_1 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_5 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_9$

$\beta_6 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_7 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_8 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_9$

$\beta_9 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$

$\beta_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_{11} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_{12} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

$\beta_{13} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

Case iii: When failure rates follow exponential time distribution

Exponential distribution is nothing but a particular case of Weibull distribution for p=1 and is very useful for practical problems purpose. Therefore, the reliability of considered system as a whole at an instant’s’ is expressed as:

$$R_{SE}(t) = \sum_{i=1}^{14} \exp.\{-\alpha_i t\} - \sum_{j=1}^{13} \exp.\{-\beta_j t\} \qquad \dots (25)$$

Where, α_i and β_j ’s have mentioned earlier.

Also, an important reliability parameter, viz; M.T.T.F., in this case is given by

$$\text{M.T.T.F} = \int_0^\infty R_{SE}(t)dt$$

$$= \sum_{i=1}^{14}\left(\frac{1}{\alpha_i}\right) - \sum_{i=1}^{13}\left(\frac{1}{\beta_j}\right) \quad \ldots (26)$$

7. NUMERICAL EXAMPLE

For a numerical computation, setting:

(A) $\lambda_i(i = 1,2.....9) = 0.001,\ p = 2\ and\ \mathrm{t} = 0,1,2--$ in equation (24);

(B) $\lambda_i(i = 1,2.....9) = 0.001,\ \ and\ \mathrm{t} = 0,1,2--$ in equation (25);

(C) $\lambda_i(i = 1,2--9) = 0,0.1......1.0$ In equation (26);

one can compute the table-1 and 2 .The corresponding graph have been shown through fig-3 and 4 respectively.

t	$R_{SW}(t)$	$R_{SE}(t)$
0	1	1
1	0.998996	0.998996
2	0.995929	0.997982
3	0.990644	0.99696
4	0.982896	0.995929
5	0.972369	0.994889
6	0.958707	0.993841
7	0.941541	0.992784
8	0.920532	0.991718
9	0.895403	0.990644
10	0.865976	0.989562

Table-1

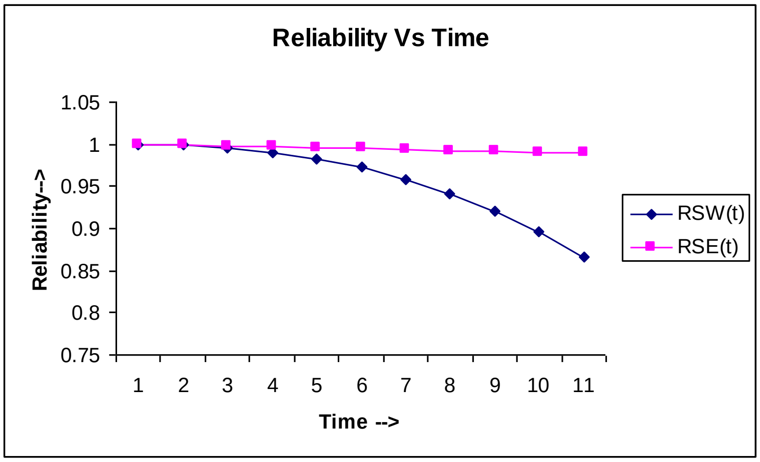

Fig-3-

λ	MTTF
0	∞
0.1	3.936507937
0.2	1.968253968
0.3	1.312169312
0.4	0.984126984
0.5	0.787301587
0.6	0.656084656
0.7	0.562358277
0.8	0.492063492
0.9	0.437389771
1	0.393650794

Table-2

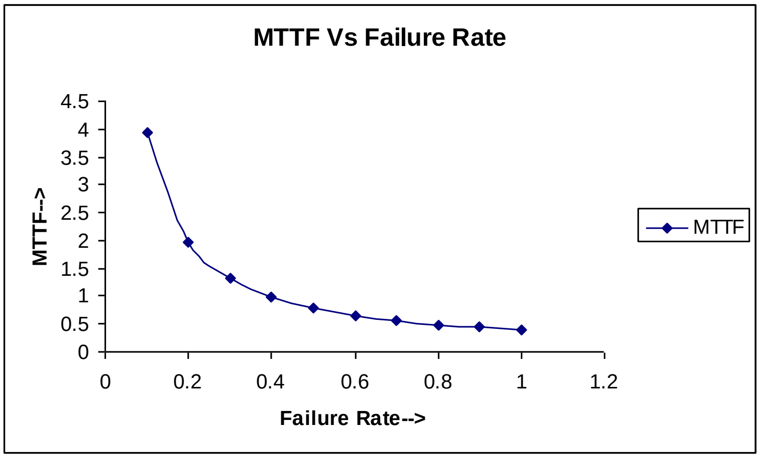

Fig-4

8. RESULTS AND DISCUSSION

In this paper, the author has considered a milk powder plant for analysis of various reliability parameters by employing the Boolean function technique & algebra of logics,

Table 1 computes the reliability of the system with respect to time when failures rates follow exponential and Weibull time distributions. An inspection of graph 'Reliability Vs Time" (fig:3) reveals that the reliability of the complex system decreases approximately at a uniformly rate in case of exponential time distribution, but is decreases very rapidly when failure rates follow Weibull distributions.

Table 2 and graph "MTTF V/S Failure Rate"(fig4) yields that MTTF of the system decreases catastrophically in the beigning but later it decreases approximately at a uniform rate.

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